ATOMIC PHYSICS [ATOMIC SPECTRA]

SYLLABUS:

Introduction –Drawbacks of Bohr's atomic model- Sommerfeld's elliptical orbits-relativistic correction (no derivation).Vector atom model and Stern-Gerlach experiment - quantum numbers associated with it. L-S and j- j coupling schemes.Zeeman effect and its experimental arrangement.

Raman effect, hypothesis, Stokes and Anti Stokes lines. Quantum theory of Raman effect. Experimental arrangement – Applications of Raman effect.

INTRODUCTION:

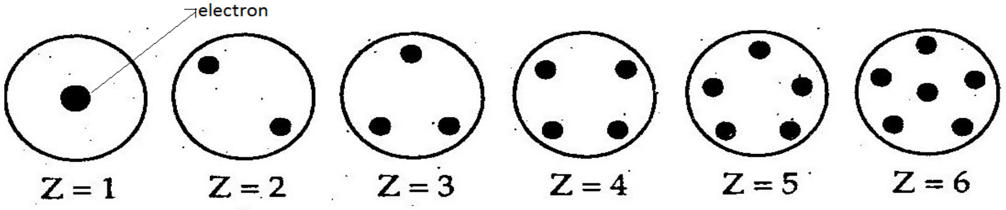
In nineteenth century, electron was first discovered by J.J. Thomson. It is negatively charged particle. But the atom as a whole is electrically neutral. Hence it is clear that the atom must contain +vely charged particle to balance the negative charge of the electron. This was confirmed by the discovery of the proton. Thomson measured the ratio of e/m value. The charge of electron was measured by Millikan oil drop experiment. The mass of the electron is found by dividing charge e by the ratio e/m. Thomson showed that the mass of the electron is of the order 1/2000th of mass of hydrogen. But they do not know how the +ve and –ve charges are distributed in an atom.

Scientists proposed several theories from time to time regarding the atomic structure. They are

(1) Thomson's plum pudding model.
(2) Rutherford's nuclear model.
(3) Bhor's model.
(4) Sommerfield's relativistic model
(5) Vector model and
(6) Wave mechanical model.

(1) THOMSON'S PLUM-PUDDING MODEL:

According to Thomson the +ve charge inside an atom is uniformly distributed through out the entire atom. The negatively charged electrons were imagined to be embedded at different places inside the continuous +ve charge as shown in the figure.

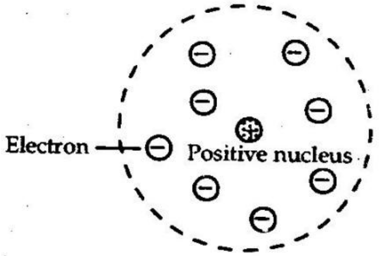


Thomson's plum - pudding model of stable atoms

DRAW BACKS: According to Thomson, the emission of radiation occurs when the stable distribution of electrons is disturbed and electrons start vibrating like oscillators. Hence emission spectrum consists of a single line of 1400A^o.But we know that the atomic spectra of hydrogen consists of several lines.

(2) RUTHERFORD'S NUCLEAR MODEL:

According to Rutherford's model the atom consists of a tiny massive, +vely charged nucleus surrounded at a relatively great distance by electrons as shown in the figure (from the experiment scattering of alpha particles by thin metal foil).In this model the electrons can't remain stationary because of the electrostatic force of attraction by the positively charged nucleus. Hence the electrons were revolving around the nucleus. The electrostatic force of attraction provides necessary centripetal force for revolving of electrons. Hence there exist dynamically stable orbits.



Rutherford's Nuclear atom

DRAW BACKS: Here there is no restriction about the radius of the electron orbit. According to classical electromagnetic theory any charged particle having accelerated motion must emit energy radiation. Hence the electron should radiate energy continuously .so the electron looses energy and spirals down and crashes in to the nucleus within 10^{-8} s .This gives continuous spectrum but not line spectrum.

(3) BHOR'S THEORY:

Inorder to overcome the problem of instability of Rutherford's planetary model of atom, Bhor proposed that an electron revolving in an orbit such that its angular momentum is quantalised will not radiate energy and the orbit will be stable. If the electron moves from one orbit to another orbit then it emits or absorbs radiation.

DRAW BACKS OF BHOR'S THEORY:

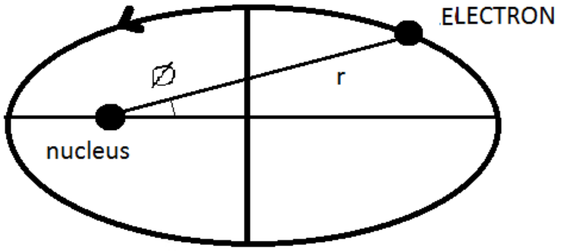
(1) The theory could not explain the spectra of atoms more complex than hydrogen.
(2) The theory does not give any information regarding the distribution and arrangement of electron in an atom.
(3) The theory does not explain the experimentally observed variations in intensity of the spectral lines of an element.
(4) This theory can't be used to calculate about transitions from one level to another.
(5) This theory can't be used for the quantitative explanation of chemical bonding.
(6) The theory fails to give correct result when an electric or magnetic field is applied to the atom.
(7) It failed to explain the fine structure of spectral lines.

According to Bhor's theory of circular orbit, we get only a single line when electron jumps from a higher orbit to a lower orbit but experimental results showed that the $H_\alpha, H_\beta, H_\gamma$ lines of hydrogen atomic spectra are not single lines. For example H_α consists of five lines. This is called fine structure of spectral line.

(4) SOMMERFELD'S RELATIVISTIC MODEL:

In 1921 sommerfeld modified Bhor's model by introducing the idea of "motion of electron in an elliptical orbits and taking into consideration the variation of mass with velocity".

ELLIPTICAL ORBIT:



According to sommerfeld, the electron moving around the nucleus in an elliptical orbit with the nucleus at one focus.In the elliptical orbit the position of the electron at any time may be fixed by two co-ordinates r and ϕ where r is the radius vector and ϕ is the angle between the radius vector and the major axis of the ellipse which is called azimuthal angle. Consider an electron of mass m and linear tangential velocity v revolving in the elliptical orbit. The tangential velocity of the electron can be resolved into two components. One along the radius vector called radial velocity and the perpendicular to radius vector called the transverse velocity. Corresponding to those velocities the electron has two momenta called as radial momentum and the transverse momentum or azimuthal momentum.

Sommerfeld introduced the two quantum numbers $n_r and\ n_\phi$ called as radial quantum numbers and azimuthal quantum numbers. Here $n_r + n_\phi = n$, where n is called principal quantum number.

Corresponding to n=1 there are two possibilities i.e., $n_r = 0, n_\phi = 1 and n_r = 1, n_\phi = 0$.when $n_\phi = 0$ the ellipse is converted into a straight line and the electron pass through the nucleus twice during every period. This type of motion is not possible. Hence, the lowest possible value of $n_\phi = 1$.

Therefore if n = 1 then $n_\phi = 1$[s orbit]

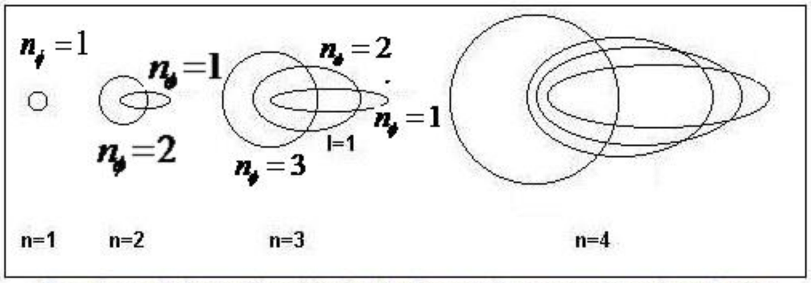
If n = 2 then [p orbit]

$$n_\phi = 2 \quad (circularobit)$$
$$n_\phi = 1 \quad (ellipse)$$

If n = 3 then [d orbit]

$$n_\phi = 3 \quad (circularobit)$$
$$n_\phi = 2 \quad (ellipse)$$
$$n_\phi = 1 \quad (ellipse)$$

These are shown in the figure.



The allowed electronic orbits for the four main quantum numbers by the Bohr-Sommerfeld model.

According to sommerfeld each momentum component is separately quantized as

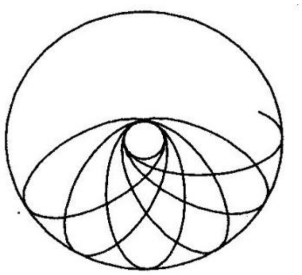
$$\oint P_r dr = n_r h$$
$$\oint P_\phi d\phi = n_\phi h$$

Where, $P_r and P_\phi$ are the components of linear momentum P of the electron along radial direction and transverse direction. $n_r and n_\phi$ are the integers, and $\oint$ indicates that the integration is to be carried over one period of the motion.

SOMMERFELD RELATIVISTIC CORRECTION:

The velocity of the electron moving in an elliptical orbit varies at different parts of the orbit. The velocity is maximum when the electron is nearest to the nucleus and minimum when it is farthest from the nucleus. Therefore the effective mass of the electron will be different at different parts of its orbit. Hence the curvature of elliptical path is not exactly the same in two positions and hence the ellipse is not complete one. Sommerfield has shown that the path of the electron is not closed ellipse but a complicated curve known as rosette-a processing ellipse as shown in the fig.

In an elliptical orbit, the velocity of the electron becomes very large when it passes close to the nucleus. Now its mass also increases. This effect makes the energy of the electron in an elliptical orbit greater than in a less elliptical orbit. Hence the different n_ϕ orbits with a given n have slightly different energies.



The energy of an electron in a particular state characterized by quantum number n and n_ϕ, after relativistic corrections can be written as

$$E_n = -\frac{mz^2e^4}{8\in_0^2 c^2h^2}\left[1+\frac{\alpha^2z^2}{n}\left[\frac{1}{n_\phi}-\frac{3}{4n}\right]\right]$$

Where $\alpha = \frac{e^2}{z\in_0 ch} \cong \frac{1}{317}$

α is called sommerfeld's fine structure constant.

From the above equation it is clear that the energy of an electron depends not only on the principal quantum number.

For n=1, n_ϕ=1 only one energy level exist

For n=2, n_ϕ=1, 2 only two energy level exist.

For n=3, n_ϕ=1, 2, 3 only three energy level exist

Here the energy difference between these levels is very small and the frequency of radiation emitted in the transition of electron from n = 3 to n = 1 will be slightly different. This is the reason why a single line of hydrogen spectrum should appear as a group or closely associated lines.

VECTOR ATOM MODEL

DRAWBACKS OF BOHR AND SOMMERFELD ATOMIC MODEL

(1) Bohr's theory was in capable of explaining the fine structure of the spectral line.
(2) In the case of complex atoms, Bhor's theory fails to calculate the energy of the system frequencies of radiation emitted.

(3) Sommerfeld modification was not predicting the correct number of observed fine structure of hydrogen.
(4) Both models could not explain the distribution and arrangement of electrons in atoms.
(5) Both models could not explain the variation of intensities in the spectral line.
(6) Both models could not explain both stark and Zeeman Effect.

CONCEPT OF VECTOR ATOM MODEL:

This model explains the concept of spacial quantization and spinning electron.

(1) Space quantization:

According to Bhor's theory, the moving electron in a circular orbit has only one degree of freedom. According to sommerfield's elliptical orbits, the electron possess two degrees of freedom, one due to n_r and the other due to n_ϕ.

In general the motion of electrons in an atom is three dimensional and possesses three degrees of freedom. Hence an additional quantum number is required to describe the state of electron in the atom in which the orbits are assumed to be quantized both in magnitude and direction. The orientation of elliptical orbit in three dimensions does not alter the original sommerfeld orbit. Thus the introduction of spacial quantization makes the orbit vector quantized.

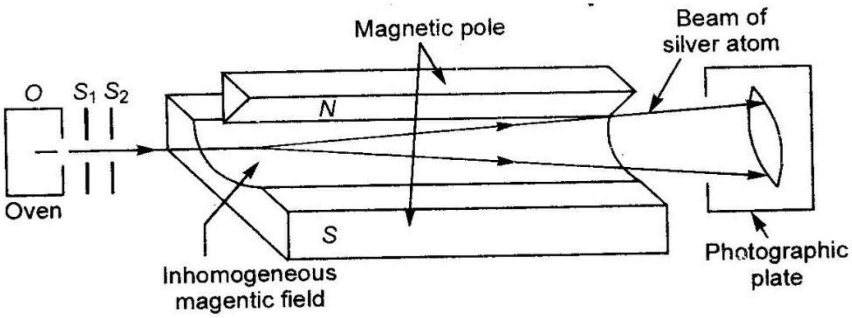
According to the rule of space quantization, the electron orbit can only set itself in certain discrete positions with respect to the field direction only. This is known as spacial quantization and was proposed by sommerfeld.

SPINNING OF ELECTRONS:

To explain many lines consist of a group of lines closed to each other Uhlenback and Goudsmit proposed the hypothesis of electron spin. According to this hypothesis, the electron revolves not only in an orbit but also about its own axis. Hence electron has orbital motion and spin motion. According to quantum theory, the spin motion should be quantized. Hence a new quantum number called as spin quantum number is introduced. The orbital and spin motions are considered as quantized vectors and hence the atom model is known as vector atom model.

STERN-GERLACK EXPERIMENT:

In Stern-gerlack experiment a beam of silver atoms is produced from an electric oven and passed the beam through slits s_1 and s_2



Stern-Gerlach experiment.

This type of silver atom is passed through an inhomogeneous magnetic field as shown in the fig. The magnetic field is of much greater intensity near the knife edge than any where else in the gap. A photo graphic plate records the configuration of the beam. The whole arrangement is enclosed in a highly evacuated glass vessel.

In the absence of magnetic field, we get a photograph of a narrow strip. In the presence of the inhomogeneous magnetic field the strip splits up into two components as shown in the fig (b) as a result of electron spin.

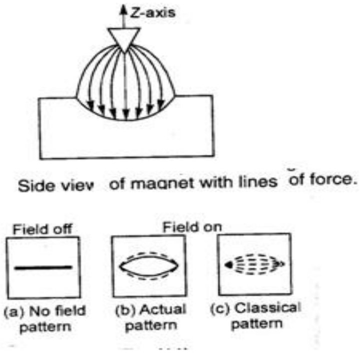


Fig. b

EXPLANATION:

Atomic number of Ag is 47.According to Pauli's exclusion principle all the inner shells and sub shells are completely filled except the outermost electron in 5s state. Thus Ag is monovalent element. So 5s electron is responsible for the magnetic moment of the atom. When all silver atoms possessing a magnetic moment μ_J pass through the inhomogeneous magnetic field, they experience different amount of force in vertical direction depending upon their orientation. If the magnetic moment μ_J can have all possible orientations, then the beam of silver atoms will be spread out into a broad continuous band on emerging from the magnetic field. But experimentally only two narrow strips are obtained on the photographic plate. Therefore the predictions of classical physics are not correct. According to quantization rules

Angular momentum $J = \sqrt{j(j+1)}.\dfrac{h}{2\pi}$

There are 2j+1 possible orientations of J

The Stern-gerlack experiment shows that 2j+1 = 2 or j = 1/2

Thus $J = \dfrac{\sqrt{3}}{2}.\dfrac{h}{2\pi}$

We know that the angular momentum J of silver atoms is entirely due to spin of its valence electrons. Hence we conclude that the electron has a spin angular momentum

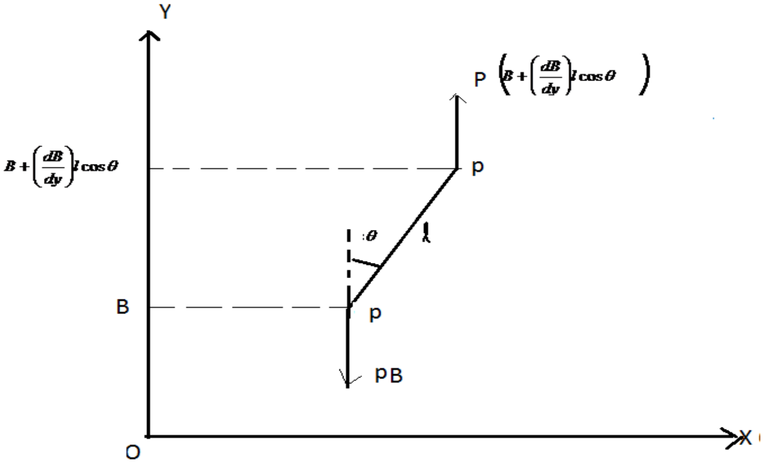
$S = \sqrt{s(s+1)}.\dfrac{h}{2\pi}$. Where s = 1/2

Hence Stern and Gerlach found that the initial beam split into two distinct parts corresponding to two opposite spins.

AN EXPRESSION FOR THE AMOUNT OF DEVIATION PRODUCED:

As shown in the fig non-uniform magnetic field is taken along y-axis .The field gradient dB/dy is +ve. Let the atomic magnet with magnetic moment M, pole strength P and length l inclined at an angle θ with the field direction. If the field strength at one pole is B then the pole strength at other pole will be $B+\left(\frac{dB}{dy}\right)l\cos\theta$

The extra force F_y is given by



$$F_y = pl\cos\theta.\frac{dB}{dy} = M\cos\theta.\frac{dB}{dy} ---------(1)$$

Due to this force the atomic magnet will be displaced from its straight path in the field direction.

Let the silver atoms enters the non-uniform magnetic field with velocity v and the length of the path be L for which time taken is t. The displacement along the field direction is given by

$$d_y = \frac{1}{2}\alpha_y t^2$$

Here α_y is the acceleration of the atom along field direction.

$$\alpha_y = \frac{F_y}{m} \quad \text{m=mass of the atomic magnet.}$$

$$d_y = \frac{1}{2}.\frac{F_y}{m}t^2 = \frac{1}{2}.\frac{F_y}{m}\left(\frac{L}{v}\right)^2$$

$$d_y = \frac{1}{2}.\frac{M\cos\theta}{m}.\frac{dB}{dy}.\left(\frac{L}{v}\right)^2 ----------(3)$$

From the above equation we can calculate the displacement of the atom along the field direction.

QUANTUM NUMBERS ASSOCIATED WITH VECTOR ATOM MODEL:

The quantum numbers associated with each of the electron in a given atom are

(1) THE PRINCIPAL QUANTUM NUMBER (n): n = 1, 2, 3……

It tells about the energy of an electron and the size of the orbital i.e., the distance from the nucleus. The principal quantum number n is a +ve integer.

The energy of the electron is given by

$E_n = -13.6/n^2$ ev

The distance from the nucleus for Hydrogen atom is given by

$r_n = 0.529n^2\ A^0$

The principal quantum number n=1,2,3,4… are denoted by K,L,M,N,O,P,Q…The number of electrons in a shell is limited to $2n^2$. The total number of orbitals for a given n value is n^2. For a hydrogen atom with n=1, the electron is in its ground state. If the electron is in the n=2 orbital it is in excited state.

(2) ORBITAL QUANTUM NUMBER (*l*): *l* = 0, 1, 2, 3, .., n-1

This quantum number defines the shape of the orbital occupied by the electron and the orbital angular momentum of the electron. The secondary quantum number divides the shells into smaller groups of orbitals called sub shells. The orbitals corresponding to l=0, 1, 2, 3… are denoted as s(sharp), p(principal), d(diffuse), f(fundamental)…

If n=1, l=0 there is only one possible orbital(s-orbital)
n=2, l=0, 1 there are two possible orbitals(s, p)
n=3, l=0, 1, 2 there are three possible orbitals(s, p, d)

The orbital angular momentum $P_l = \sqrt{l(l+1)}.\frac{h}{2\pi}$

The orbital quantum number *l* is similar to the azimuthal quantum number n_P of sommerfield theory. The two are related by $l = (n_\phi - 1)$

(3) SPIN QUANTUM NUMBER (s):

Spin quantum number specifies the orientation of the spin axis of an electron. The magnitude is always half. Since, the electron can spin clockwise or anti clockwise, there are two values 1/2,-1/2 depending upon clock wise or anti clockwise. The angular momentum P_s is given by $P_s = \sqrt{s(s+1)}.\frac{h}{2\pi}$

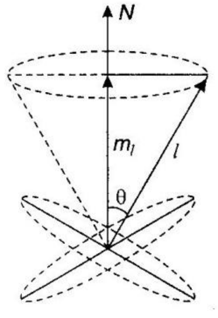
(4) TOTAL ANGULAR MOMENTUM (j):

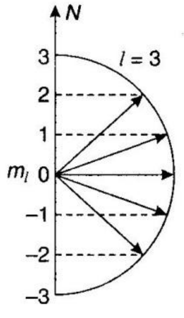
This quantum number represents the resultant angular momentum of the electron due to both orbital and spin motions. This can be expressed as j= (*l*+1/2).plus sign is used when s is parallel to *l* and minus when s is anti-parallel to *l*. the total angular momentum of the electron is

$$P_j = j.\frac{h}{2\pi}$$

(5) MAGNETIC ORBITAL QUANTUM NUMBER (m_l):

We know that due to space (direction) quantization, the orbital angular momentum can have orientations in space such that its projection on a fixed direction can vary from –*l* to +*l* in steps of one unit. This projection of *l* in the field direction is known as magnetic quantum number. m_l= *l*, (*l*-1), (*l*-2),…1, 0, -1,.., (*l*-2), (*l*-1), -*l*





In the above figure the different values of m_l = (-3,-2,-1,0,1,2,3)are shown for *l*=3. There are (2*l*+1) allowed orientations of the m_l vector.

(6) MAGNETIC SPIN QUANTUM NUMBER (m_s):

In an external magnetic field, the

spin angular momentum P_s can have only two possible orientations for which the projections will be having values $m_s = -1/2$ and $m_s=+1/2$. m_s can be either +1/2 or -1/2 and has no other values for a single electron

(7) MAGNETIC TOTAL ANGULAR MOMENTUM QUANTUM NUMBER (m_j):

Its numerical values are the projection of the total angular momentum vector j on the field direction. Since j can have half integral values, m_j can also assume half integral values. The possible values of m_j are –j, (-j+1)…,1/2, 3/2, 5/2.,…,(j-1), j excluding zero for single electron.

The permitted orientations of j are (2j+1) and hence m_j can have only (2j+1) values from +j to -j excluding zero. This explains the splitting of spectral lines in a strong magnetic field.

COUPLING SCHEMES:

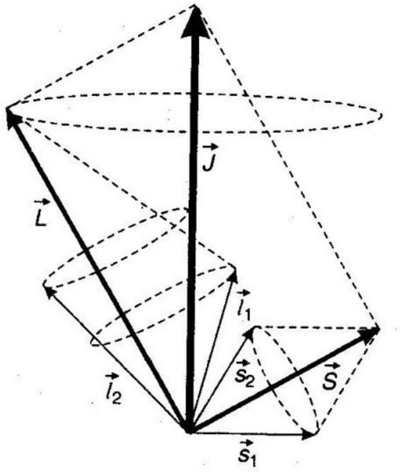
When we consider a number of electrons, the total angular momentum would depend upon the coupling (interactions) between orbital and spin angular momenta. Generally two types of coupling known as Russell-Saunders or L-s coupling and j-j coupling occurs.

(1) L-S COUPLING: In this coupling all the orbital angular momentum *l* of the electrons combine to form a resultant vector L and all the spin angular momentum vectors s combine to form a resultant vector S. now the vector Land S combine to form a vector J which represents the total angular momentum of the atom. This can be represented as

$L= (l_1+l_2+\ldots)$

$S= (s_1+s_2+s_{3\ldots})$

$J=L+S$



Showing L-S Coupling

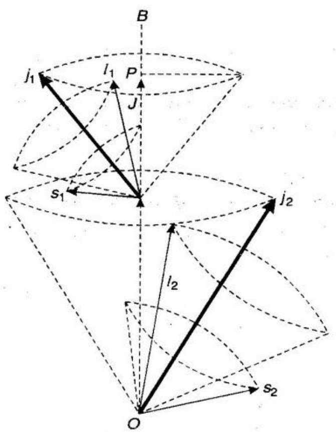
Here L is always an integer i.e., 0, 1, 2, 3…. the value of S depends upon the number of electrons and direction of their spin vector. In the case of two electrons, the value of s is one or zero. In the case three electrons, the value of S may be 3/2 or 1/2 . In the case four electrons the value of S may be 2, 1 or zero. Thus S is an integer for an even number of electrons and half integer for an odd number of electrons.

Now J=0, 1, 2, 3….. When S is integer

J=1/2, 3/2, 5/2…when S is half integer

J must always be +ve and never –ve.

(2) J-J COUPLING:



In j-j coupling, each electron is considered separately and its total angular momentum j is obtained by the relation j=*l*+s. then the total angular momentum J of the atom would be vector sum of all the individual j vectors of the electrons.

Hence $j_1= (l_1+s_1)$, $j_2= (l_2+s_2)$, $j_3= (l_3+s_3)$….

$J=j_1+j_2+j_3+\ldots$

$J=\sum j$

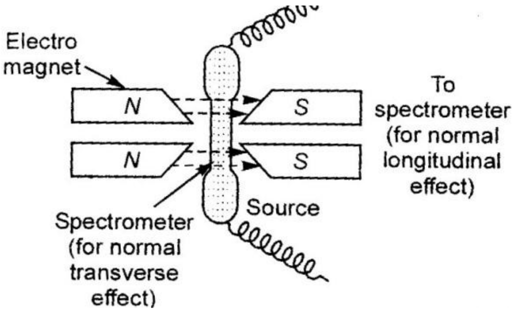
In most of the known cases L-S coupling is effective.

ZEEMAN EFFECT:

In 1986 Prof. Zeeman discovered that when a source of radiation, giving line spectrum is placed in a magnetic field; the spectral lines are split up into a number of component lines, symmetrically distributed about the original line. This is known as Zeeman Effect.

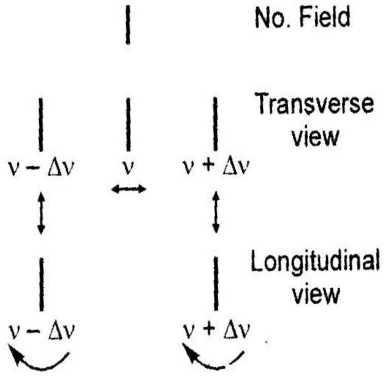
If the magnetic field is very strong, each spectral line is split up into two components in the longitudinal view and three components in transverse view. This is normal Zeeman Effect. When the magnetic field is weak, each line splits into more than three components. This is known as Anomalous Zeeman Effect.

EXPERIMENTAL ARRANGEMENT:



The experimental arrangement is as shown in the figure. The source of radiation such as sodium or mercury arc is placed between the pole pieces of a powerful electromagnet. It is in conical shape and holes are drilled along the length so that the light from the source can pass through it. Hence the spectrum can be observed along the direction of magnetic field. The spectral lines are observed with a high resolving power spectroscope. The spectral lines may also be observed in a direction perpendicular to the magnetic field.

In the normal Zeeman Effect, when the light is observed perpendicular to the direction of magnetic field, three components are observed. One in the same position having frequency υ and two components being on either side of the central line $\upsilon+\Delta\upsilon$ and $\upsilon-\Delta\upsilon$. When the light is observed along the direction of magnetic field the same two additional lines with frequencies $\upsilon+\Delta\upsilon$ and $\upsilon-\Delta\upsilon$ are observed while the central line is missing.



APPLICATION OF ZEEMAN EFFECT:

Using Zeeman Effect the specific charge e/m$_o$ of the electron can be calculated. When the source is placed in the magnetic field, the change in frequency dυ of the spectral line is given by

$$d\upsilon=\frac{eB}{4\pi m_o}$$

If λ be the wavelength, then $\upsilon=\frac{c}{\lambda}, d\upsilon=\left(\frac{-c}{\lambda^2}\right)d\lambda$

$$d\lambda=\frac{\lambda^2}{c}d\upsilon=\frac{\lambda^2}{c}.\frac{eB}{4\pi m_o}=\frac{eB\lambda^2}{4\pi m_o c}$$

$$\therefore \frac{e}{m_o}=\frac{4\pi c}{Bn}.\frac{d\lambda}{\lambda^2}$$

MOLECULAR SPECROSCOPY

INTRODUCTION:

When electromagnetic radiation is allowed to fall on an atomic system, the electrons of that atom absorb the photon energy of incident radiation and goes to excited state. After the life time is completed the atoms come to the excited state by emitting the energy equal to the energy difference of the two states. The emitted photon is also having a single wavelength or frequency and the spectrum is known as line spectra or atomic spectra.

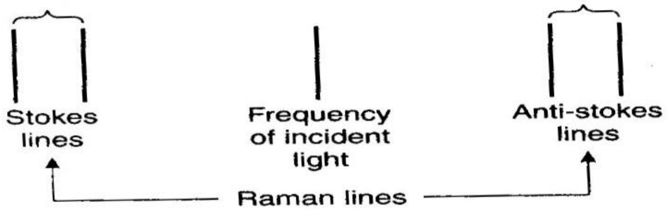
MOLECULAR SPECTRA:

This spectroscopy deals with the interaction of electromagnetic radiation with molecules. The molecule is also having ground state and excited states. If the incident radiation on molecule is less than the energy difference of ground state and excited state, then the molecule rotates about its centre of mass giving rise to rotational energy levels. If the incident radiation is having still more energy than the above case, the nuclei of molecule vibrate along the inter-nuclear axis giving rise to vibrational energy levels. The electronic transition in molecule takes place in the ultra violet or visible region of the electromagnetic spectrum. $E_{total} = E_{rot} + E_{vib} + E_{ele}$

RAMAN EFFECT:

When monochromatic beam of light of frequency γ_o is passed through a dust free transparent substance the light is scattered in all directions. Most of the scattered radiation has the frequency γ_o as that of incident radiation. This was observed by Raman in 1928 and the effect is known as Raman Effect.

If $\gamma_i and \gamma_s$ are the frequencies of the incident and scattered radiation the difference in frequency $\Delta\gamma = \gamma_i - \gamma_s$ is known as Raman shift. The Raman spectra of ccl_4 are shown in the fig.



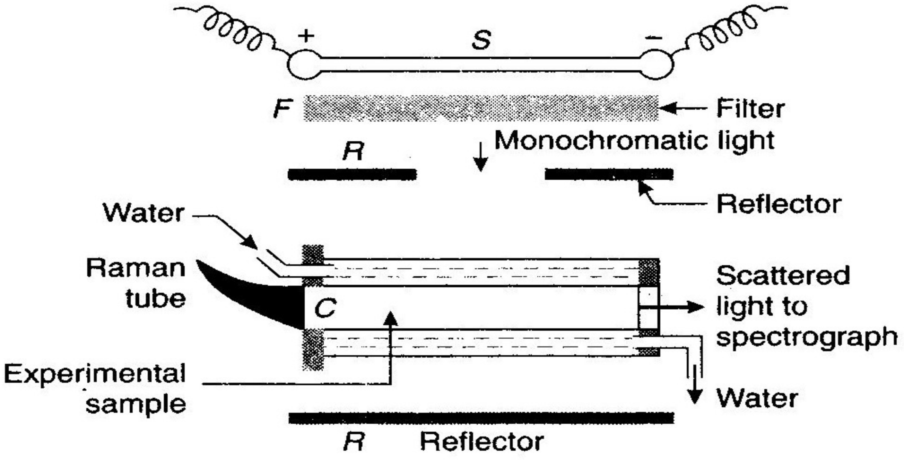
Raman lines

The lines on the lower frequency side are called stokes lines and those on higher frequency side are known as anti-stokes lines of unchanged frequency are due to Rayleigh scattering stokes lines are always more intense than anti-stokes lines.

CHARACTERISTICS OF RAMAN EFFECT:

(1) The frequencies of Raman lines depend on the frequency of incident light.
(2) The displacement of Raman lines from the original line depend on the nature of scattering substance.
(3) The anti stokes lines are weaker than stokes-lines.
(4) Raman Effect is purely a molecular phenomenon.
(5) The rotational or vibrational state of the molecule is changed due to scattering of light by it.

EXPERIMENTAL ARRANGEMENTS FOR RAMAN SPECTROSCOPY:



Experimental arrangement for Raman spectrum

In Raman spectrometer the sample is irradiated with mono-chromatic light and is observed at right angles to the incident radiation. The experimental arrangement is as shown in the fig. it consists of the following parts.

(1) source (2) Raman tube (3) spectrograph

Raman tube C is filled with the liquid which is to be investigated. It is made of glass and is 1-2 cm in diameter and 10-25 cm long. One end of the tube is drawn like a horn and blackened outside to provide a suitable black background. The other end of the tube is closed with an optically plane glass plate. The scattered light emerges through the window W. the Raman tube is surrounded by water to cool the sample.

S is a mercury arc lamp which acts as a source of light. This light is filtered by nickel oxide glass filter to get a mono-chromatic the light is then allowed to pass through an opening in a metallic reflector which then falls on Raman tube. The experimental sample is placed inside the Raman tube. The scattered beam emerges from the flat end of Raman tube and is examined by means of a spectrograph.

When the sample is a gas, it is enclosed in a tube at a pressure of a few cm of Hg.
In the case of solids, powders are used. They are mixed either with liquid paraffin or any other solvent.

QUANTUM THEORY OF RAMAN EFFECT: According to quantum theory, the incident radiation of frequency υ can be treated as consisting of a stream of particle called photons having energy $h\upsilon$ .Interaction of incident radiation with molecules of the transparent substance can be treated as collision between photons and molecules of the substance.

Suppose a molecule of mass m in the energy state E_p is moving with velocity v and is colliding with a light photon of energy $h\upsilon$. Suppose this molecule undergoes a change in its energy state as well as in its velocity. Let this new energy state be E_q and the velocity is V'. From the principle of conservation of energy, we have

$$E_p + \frac{1}{2}mV^2 + h\upsilon = E_q + \frac{1}{2}m(V')^2 + h\upsilon'$$

Since the change in velocity of the molecule is negligible the above equation can be written as

$$E_p + h\upsilon = E_q + h\upsilon'$$

$$or \quad \upsilon' = \upsilon + \frac{E_p - E_q}{h}$$

$$or \quad \upsilon' = \upsilon + \Delta\upsilon$$

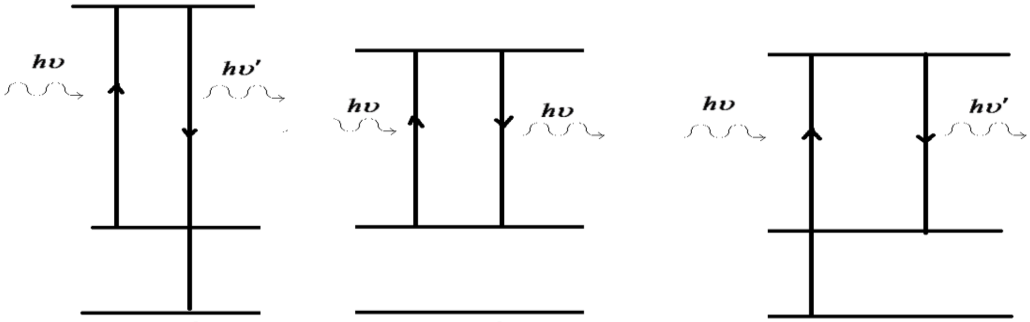
From this equation three different situations can occur. They are

1) If $E_p = E_q$, then $\upsilon' = \upsilon$ and this refers to unmodified line where the molecule simply deflects the photon without receiving energy from it.

2) If $E_p > E_q$, then $\upsilon' > \upsilon$ and this refers to anti-stokes lines. Here the molecule was previously in the excited state and it handed over some of its intrinsic energy to the incident photon. Hence the photon has greater energy.

3) If $E_p < E_q$, then $\upsilon' < \upsilon$ and this refers to stokes lines. Here the molecule absorbed some energy from the photon. Hence the photon has lower energy.

The three different situations are shown in the following figure.



$\lambda < \lambda'$ **Anti stokes line**

$\lambda = \lambda'$ **Rayleigh line**

$\lambda > \lambda'$ **stokes line**

For transition between the energy levels, we get different spectral lines.

<u>APPLICATIONS OF RAMAN EFFECT</u>:

(1) To study the structure of crystal and compounds.
(2) To know the number of atoms in a molecule, their relational arrangement, relative mass and chemical bonds between them.
(3) To study the composition in plastic, mixtures etc.
(4) To decide about single, double, triple bonds.
(5) To study the spin and statistics of nuclei.
(6) To study the binding forces between the atoms or groups of atoms in crystals.
(7) The study the rotational or vibrational energy levels of homo nuclear molecules as example nitrogen, oxygen etc...
(8) To determine many of the structural parameters of the molecule.

Crystal Structure

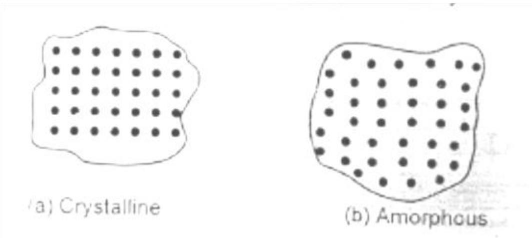
{Syllabus: Amorphous and crystalline materials, unit cell, miller indices, reciprocal lattice, types of lattices, diffraction of x-rays by crystals, bragg's law, experimental techniques, laue's method and powder diffraction method.}

INTRODUCTION: Elements and their compounds are found in three states.
They are 1) Solids 2) Liquids and 3) Gases. We know that some materials are strong, some materials are week and some are good conductors of heat and electricity, some materials are bad conductors of heat and electricity, some materials are magnetic and some are on magnetic and so on.

Solids are divided into two broad categories. They are crystalline solids and amorphous.

Crystalline solids: If the atoms or molecules in a solid are arranged in regular fashion then it is known as crystalline. The atoms are arranged in an orderly three dimensional array that is repeated through the structure.
The metallic crystals are Ag, Cu,Al,Mg etc..
The non metallic crystals are C,Si,Ge etc..

Amorphous solids: When the atoms or molecules in a solid are arranged in an irregular fashion then it is known as amorphous solids.
Ex: Glass, Plastic, Rubber etc..



Distinction between crystalline and amorphous solids:

S.No.	Crystalline solids	Amorphous solids
1	Crystalline solids have a regular arrangement of particles	The amorphous solids have a completely random particle arrangement
2	Crystalline solids have different physical properties (thermal conductivity refractive index etc..) in different directions. (Anisotropic)	They have their physical properties same in all directions. (Isotropic)
3	The cooling curve of crystalline substance has breaks	The cooling curve for amorphous substance is smooth
4	All the bonds in crystalline solids are equally strong due to the symmetry	All the bonds in amorphous solids are not equally strong
5	In case of crystalline substance, the melting point is very sharp	The amorphous substance does not have the sharp melting point.

Unit cell: The smallest geometric figure the repetition of which gives the actual crystal structure is known as unit cell.

Miller indices: The three smallest possible integers which have the same ratios as the reciprocals of the intercepts of the plane concerned on the three axes is known as miller indices.

Or

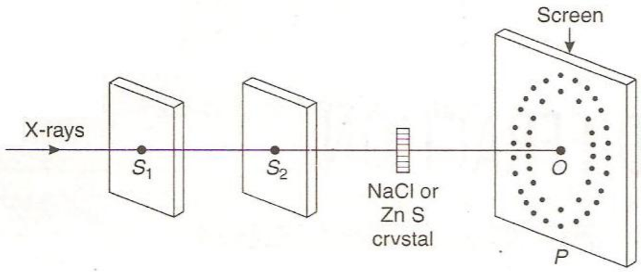
The reciprocals of the intercepts which the plane makes with the axes when reduced to smallest numbers is known as miller indices.

Reciprocal lattice:

Types of lattices:

X-rays : In 1895 Rontgen discovered X-rays, but he don't know the physical nature of X-rays. To test the nature of X-rays, a number of experiments were performed but without success. To test the wave nature of X-rays, it is essential that they must produce diffraction and interference patterns. We know that to obtain diffraction pattern, the spacing between the lines ruled on the grating should be of the order of the magnitude of the wavelength of the light used. A s X-rays are of shorter wavelength, a transmission grating have 40×10^6 lines per cm can only produce a diffraction pattern. It is impossible to construct such type of grating. In 1912, German physicist Laue suggested that a crystal which consists of a three-dimensional array of regularly spaced atoms can serve the purpose of grating.

DIFFRACTION OF X-RAYS BY CRYSTALS:



Friedrich and Knipping conducted X-ray diffraction experiment by using Laue suggestion. The experimental arrangement is as shown in the figure.

A narrow beam of X-rays are obtained by passing through the pin holed screens S_1 and S_2. The beam is now allowed to pass through a sodium chloride or zinc sulphide crystal.

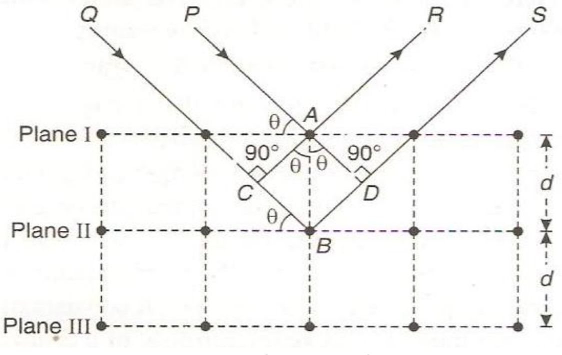
The emergent rays are made to fall on a photographic plate P. the diffraction pattern so obtained consists of a central spot at O and a series of spots arranged in a definite pattern about O. This symmetrical pattern of spots is known as Laue pattern and proves that X-rays are electro-magnetic waves. According to Bragg, the spots are produced due to reflections of some incident X-rays from the various sets of parallel crystal planes which contains large number of atoms.

From the experiment it is clear that

(1) X-rays are electromagnetic waves of extremely shorter wavelengths.

(2) The atoms are arranged in a regular three-dimensional lattice.

BRAGG'S LAW:



As shown in the fig parallel lattice planes of a crystal separated by a distance d. when a narrow beam of X-rays of wavelength λ is incident upon the planes at an angle θ.

Consider a ray PA reflected at A in the direction AR from plane 1 and another ray QB reflected at B in the direction BS. Now draw two perpendicular lines AC and BD from A. the path difference from these two rays is (CB+BD). The two reflected rays will be in phase or out of phase will depend upon the path difference. The condition of reinforcement is $\therefore CB + BD = n\lambda$

$$CB = BD = d\sin\theta$$

$$\therefore 2d\sin\theta = n\lambda \text{------------------}(1)$$

Where n=1, 2, 3….. For first order, second order… etc. The above relation is known as Bragg's law.

For first maxima $\sin\theta_1 = \frac{\lambda}{2d}$

For second maxima $\sin\theta_2 = \frac{2\lambda}{2d}$

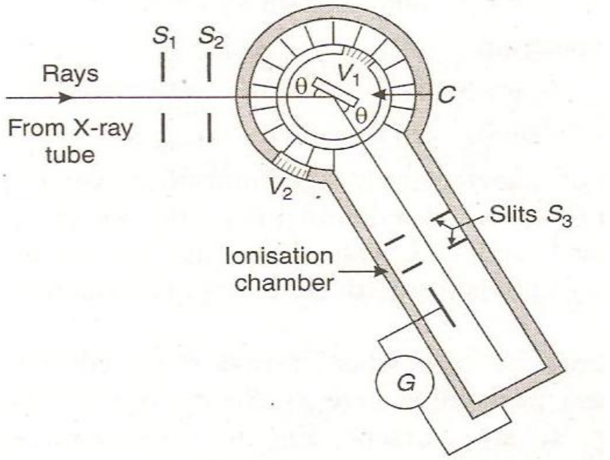
For third maxima $\sin\theta_3 = \frac{3\lambda}{2d}$

From the above relations it is clear that intensities go on decreasing as the order of the spectrum increases.

Experimental techniques:

BRAGG'S X-RAY SPECTROMETER:

As shown in the fig X-rays from X-ray tube are narrowed by passing them through slits S1 and S2 provided in lead screens. The beam is now allowed to fall on a crystal C mounted on a circular table of the spectrometer. The circular table is rotating about a vertical axis. The rotation can be noted with the help of circular scale and vernier scale V1. The reflected beam then passes through the slits S3 and enters in ionization the ionization chamber is mounted on the movable arm about the same axis of the crystal. The position of ionization chamber can be read by a second vernier V2. The turn table and ionization chamber arranged such that when the turn table rotates through an angle θ, the ionization chamber turn through 2θ. The X-rays entering the ionization chamber ionize the gas which causes a current to flow between two electrodes. This can be measured by galvanometer G. the ionization current is measured for different value of glancing angle θ. The graph is shown in the fig



Certain values of glancing angle θ, the intensity of ionization current increases abruptly. We know that $2d\sin\theta = n\lambda$

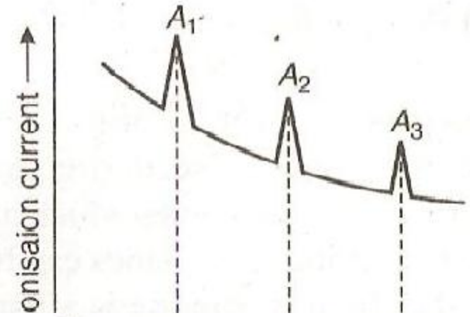
For first order spectrum n=1 $2d\sin\theta = \lambda$

For second order spectrum n=2 $2d\sin\theta_2 = 2\lambda$

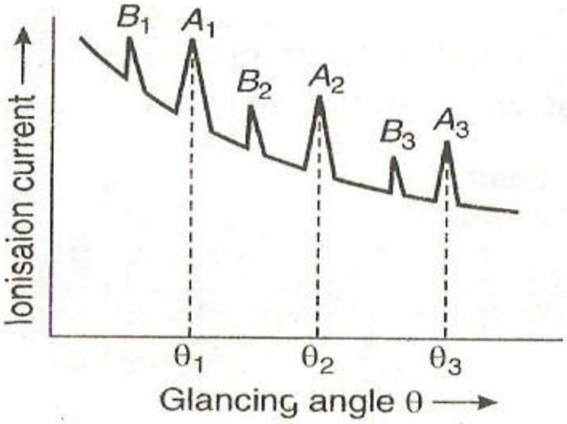
For third order spectrum n=3 $2d\sin\theta_3 = 3\lambda$

$$\therefore \sin\theta_1 = \sin\theta_2 = \sin\theta_3 = 1:2:3$$

This shows that the assumption that X-rays get reflected like ordinary light is justified.



When X-rays consists of two wavelengths $\lambda_1 and \lambda_2$ then the peak values like A_1, B_1,A_2,B_2 and A_3, B_3 are obtained as shown in the fig. the peak value A_1 ,A_2 and A_3 are corresponding to wavelength λ_1 while the other for λ_2.it is observed that



$\sin\theta_1 : \sin\theta_2 : \sin\theta_3 = 1:2:3$ and $\sin\theta_1^1 : \sin\theta_2^1 : \sin\theta_3^1 = 1:2:3$

From the observed value of θ and known value of d and n, the wavelength of X-rays λ can be calculated by using the equation $2d\sin\theta = n\lambda$

OBSERVATIONS:

(1) As the order of the spectrum increases, the intensity of reflected X-rays decreases.
(2) The ionization current does not fall to zero for any value of glancing angle θ.

BRAGG'S LAW AND CRYSTAL STRUCTURE:

In case of *KCl* structure brag obtained the maxima of reflected X-rays at glancing angles 5.22^0, 7.30^0 and 9.05^0.using the three different reflecting planes.

According to Bragg's equation $2d\sin\theta = n\lambda$

$$\therefore \frac{1}{d} = \frac{2\sin\theta}{\lambda}$$

$$\therefore \frac{1}{d_1} : \frac{1}{d_2} : \frac{1}{d_3} = \sin 5.22^0 : \sin 730^0 \sin 9.05^0$$

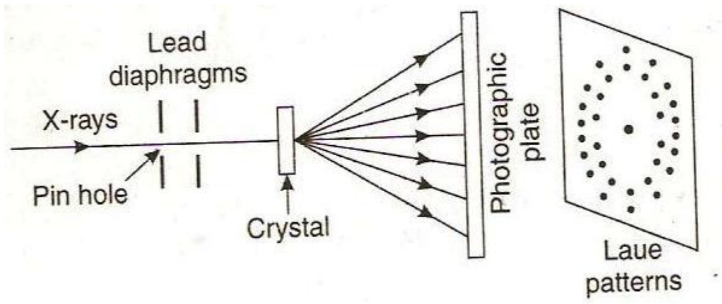
$$= 0.091 : 0.127 : 0.157$$

$$= 1 : 1.4 : 1.73$$

This result shows that *KCl* is simple cubic crystal. In this way Bragg's law can be utilized to analyze different types of crystal substances.

CRYSTALLOGRAPHY BY LAUE METHOD:

The experimental arrangement is as shown in the figure. By studying the result of this experiment we can determine the nature of X-rays and crystal substances.



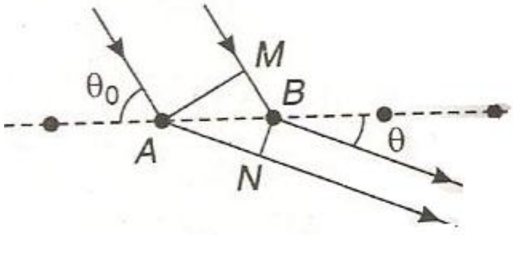
The beam of X-rays is passing through the pinholes of lead diaphragms and then it is allowed to pass through the crystal which is fixed on the circular turn table. The X-rays are diffracted and recorded on the photo-graphic plate. The diffraction pattern consists of central spot and a set of spots arranged in a definite pattern about central spot. The symmetrical pattern of spot is known as Laue pattern.

Each spot in the Laue pattern corresponds to an interference maximum for a set of crystal plane satisfying the Bragg equation $2d\sin\theta = n\lambda$

The atomic arrangement in the crystal arrangement in the crystal can be reduced from a study of the positions and intensities of the Laue spots.

THEORY:

Let us consider a line grating with atoms on the line as diffracting centers. Let 'a' be constant distance



between two successive lattice points. Suppose a monochromatic X-rays of wavelength λ are incident on the space lattice. The electrons in the atoms of the lattice points scatter X-rays in all directions. Let θ_0 the glancing angle of incidence and θ the glancing angle of diffraction as shown in the fig. The path difference between two diffracted rays is AN-BM.

Where AN, BM are perpendiculars dropped on the incident and diffracted rays. The diffracted rays will have maximum intensity if AN-BM= $n_1\lambda$

Here n_1 is an integer. From the figure AN= $AB\cos\theta = a\cos\theta$

BM= $AB\cos\theta_0 = a\cos\theta_0$

$$\therefore a(\cos\theta - \cos\theta_0) = n_1\lambda$$

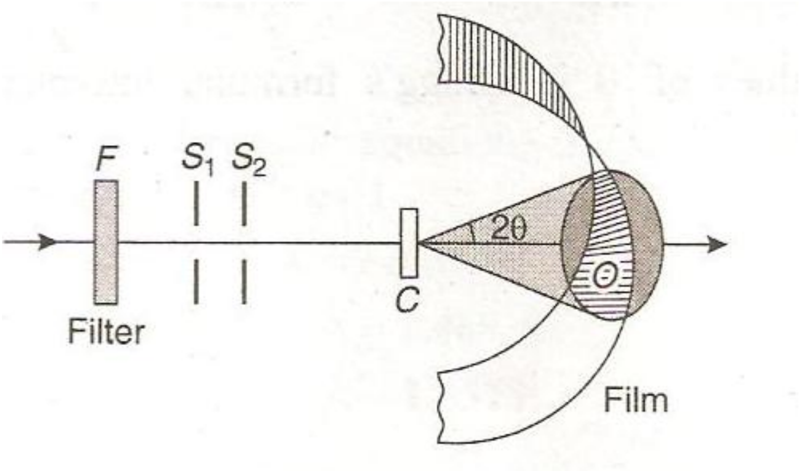
Usually $\cos\theta_0 and \cos\theta$ are denoted by $\alpha_0 and \alpha$ which is known as directional cosines of the incident and diffracted rays. $a(\alpha - \alpha_0) = n_1\lambda$

The above equation can be extended to the whole crystal considering it as a three dimensional grating which have directional cosines is $\alpha, \beta and \gamma$ then

$$a(\alpha - \alpha_0) = n_1\lambda$$
$$a(\beta - \beta_0) = n_2\lambda$$
$$a(\gamma - \gamma_0) = n_3\lambda$$

Where n_1, n_2, n_3 are integers. This means that every diffracted ray is characterized by three integral numbers n_1, n_2, and n_3 called the order numbers. These are different from the orders of the spectra which occur in Bragg's relation. These relations represent the essential features of the theory of space lattice.

CRYSTALLOGRAPHY BY POWDER METHODS:

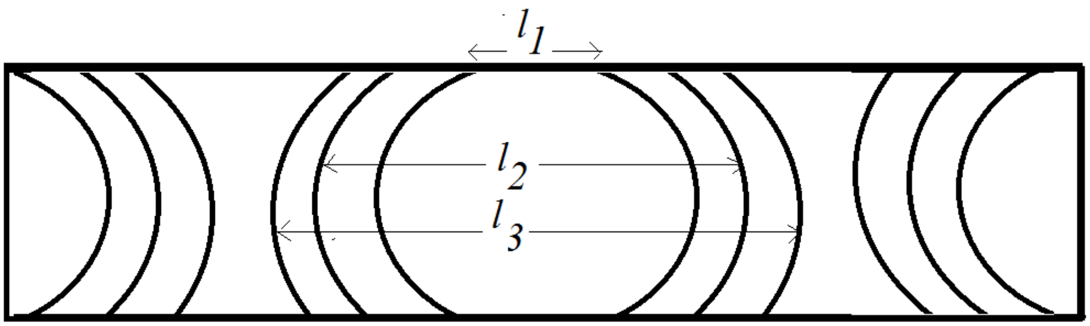


The Bragg's spectrometer can be used to diffract X-rays by using a single crystal. To know the information about the size of crystals, presence of impurities, distortion, orientation of crystals etc.We have to make use of powder method.

The experimental arrangement of powder method is as shown in the fig. X-rays from X-ray tube are allowed to pass through a filter F. which absorbs all wavelengths except one. This mono-chromatic X-rays are collimated by passing it through two slits S_1 and S_2. Then the collimated X-ray is made to fall on the powered specimen C. the specimen is located at the centre of drum shaped cassette with photographic film at the inner circumference.

In the powder, there are thousands of grains that have random orientations. Hence we expect most of the different atomic planes to lie parallel to the surface in some of the grains. Reflection takes place at these planes. For a value of θ, the beam appears at the corresponding 2θ deviation. For various values of d and n values, we have various cones of rays are obtained as shown in the fig (1).

The scattered X-rays are incident on the photographic film. The interactions of the different cones of the photographic film are series of concentric circular rings. Radii of these rings can be used to find glancing angle. Now inter planar spacing of crystalline substance can be noted.



The recorded photographic film is as shown in the fig (2). The curvature of the arcs reverses then the angle of diffraction exceeds 90^0.let l_1, l_2, l_3…..etc be the distance between symmetrical lines on the stretched photograph and D be the diameter of cylindrical film then

$$\frac{l_1}{\pi D} = \frac{4\theta_1}{360^0} or \theta_1 = \frac{90^0}{\pi D} l_1$$

Similarly $\theta_2 = \frac{90^0}{\pi D} l_2 and \theta_3 = \frac{90^0}{\pi D} l_3$

Using the above values we can calculate the interference spacing d.

MATTER WAVES &UNCERTAINTY PRINCIPLE

Matter waves, de Broglie's hypothesis - wavelength of matter waves, Properties of matter waves - Davisson and Germer experiment – Phase and group velocities.
Heisenberg's uncertainty principle for position and momentum (x and p), & energy and time (E and t). Experimental verification - Complementarity principle of Bohr.

MATTER WAVES

INTRODUCTION:

The particle is specified by (1) mass (2) velocity (3) momentum (4) energy. A wave is specified by its (1) frequency (2) wavelength (3) phase of wave velocity (4) amplitude (5) intensity. Radiation has a dual mature i.e., radiation is a wave which is spread out over space and also a particle which is localized at a point in space. Radiation sometimes behaves as a wave and at other times as a particle.

DE BROGLIE'S HYPOTHESIS OF MATTERWAVES:

DE-BROGLIE in 1924 suggested that the correspondence between wave and particle should not confined to electromagnetic radiation, but it should also be valid for material particles. Like radiation, matter also has a dual nature. A moving particle is controlled by the wave in a manner similar to that in which a photon is controlled by waves.

According to DE-BROGLIE'S hypothesis, a moving particle is associated with a wave which is known as DE-BROGLIE wave. The wave length of the matter wave is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

PROPERTIES OF MATTER WAVES:

1) If the particle is lighter, then the wavelength associated with it is greater.
2) If the velocity of the particle is smaller, then the wavelength associated with it is greater.
3) When V=0, then $\lambda = \infty$
When V=∞, then $\lambda = 0$

This shows that matter waves are generated by the motion of particles. These waves are produced whether the particles are charged particles or they are uncharged. This fact reveals that these waves are not electromagnetic waves but they are new kind of waves.
4) The velocity of matter waves depends on the velocity of matter particle.
5) The velocity of matter waves is greater than the velocity of light.
6) Waves have particle-like properties and particles have wave-like properties.
7) The wave nature of matter introduces an uncertainty in the location of the position of the particle.

EXPRESSION FOR DE-BROGLIE'S WAVELENGTH:

According to Planck's theory of radiation the energy of a photon is given by

$$E = h\upsilon = \frac{hc}{\lambda} ----------- (1)$$

According to Einstein's mass energy relation

$$E = mc^2 -------------------------- (2)$$

From esq. (1) and (2) we get $mc^2 = \frac{hc}{\lambda} or \lambda = \frac{hc}{mc^2} = \frac{h}{mc} = \frac{h}{p}$

Here p=mc, momentum associated with photon

If we consider a material particle of mass m moving with a velocity v then the wavelength associated with this particle can be expressed as $\lambda = \frac{h}{mv} = \frac{h}{p}$

SPECIAL CASES:

(1)If E is the K.E of the material particle then E = $\frac{1}{2}mv^2 = \frac{p^2}{2m} => p = \sqrt{2Em}$

DE-BROGLIE wavelength $\lambda = \frac{h}{\sqrt{2Em}}$

(2) When a charge particle carrying a charge q is accelerated by a potential difference V then its K.E is given by E = V_q

$$\therefore \lambda = \frac{h}{\sqrt{2mqV}}$$

(3) When a material particle is in equilibrium at a temperature T then E = $\frac{3}{2}KT$

$$\lambda = \frac{h}{\sqrt{2m\left(\frac{3}{2}KT\right)}} = \frac{h}{\sqrt{3mKT}}$$

VELOCITY OF DE-BROGLIE'S WAVES:

DE-BROGLIE'S wavelength $\lambda = \frac{h}{mv}$ ---------- (1)

Velocity of DE-BROGLIE'S wave $V = \upsilon\lambda$ ------ (2)

The energy of the particle is E = $h\upsilon$ ---------------------- (3)

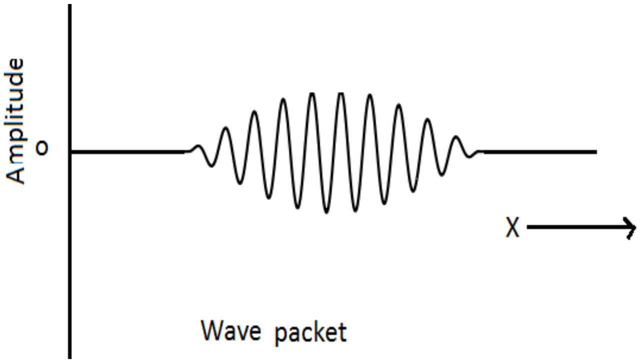
From relativity E=mc^2------------------------------------ (4)

$$\therefore h\upsilon = mc^2 \quad => \upsilon = \frac{mc^2}{h}$$

From eqns. (1) and (2) we get $V = \frac{mc^2}{h}.\lambda = \frac{mc^2}{mv\lambda}.\lambda = \frac{c^2}{v}$

THE CONCEPT OF PHASE AND GROUP VELOCITIES:

WAVE VELOCITY:



When a mono chromatic wave travels through a medium, its velocity in the medium is called as waver velocity. Consider a wave whose displacement is

Y=a sin$(\omega t-Kx)$

Here k is known as propagation constant.

The ratio of angular frequency ω to the propagation constant K is defined as wave velocity V_p.

$$\therefore V_p=\frac{\omega}{K}$$

For the wave $(\omega t-Kx)$ is the phase of wave motion

For the planes of constant phase $(\omega t-Kx)$=constant

Differentiating the above eq. with respect to t we get $\omega-K\frac{dx}{dt}=0 \Rightarrow \frac{dx}{dt}=\frac{\omega}{K}=V_p$

Thus the wave velocity is the velocity with which the planes of constant phase advance through the medium. Due to this reason, the wave velocity is also known as phase velocity.

GROUP VELOCITY:

When we consider a pulse it consists of a number of waves slightly differing in frequency from one another. The superposition of such waves is known as wave group (wave packet). When such a group travels in the medium, the phase velocities of different components are different. The observed velocity is the velocity with which the maximum amplitude of the group advances. This is called group velocity. Hence group velocity with which the energy in the group is transmitted. Group velocity $V_g=\frac{d\omega}{dK}$

PHASE VELOCITY AND GROUP VELOCITY FOR DE-BROGLIE WAVES:

The DE-BROGLIE wave $\lambda=\frac{h}{mv}$

$\omega=2\pi\upsilon,$

But $E=h\upsilon=mc^2 \Rightarrow \upsilon=\frac{mc^2}{h}$

$$\therefore \omega=\frac{2\pi mc^2}{h}$$

Also we know that $m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$

$$\therefore \omega = \frac{2\pi m_o c^2}{h\sqrt{1-\frac{v^2}{c^2}}} --------------------(1)$$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi mv}{h} = \frac{2\pi m_o v}{h\sqrt{1-\frac{v^2}{c^2}}} ---------------(2) \because \lambda = \frac{h}{mv}$$

The phase velocity of DE-BROGLIE is given by $V_P = \frac{\omega}{K} = \frac{c^2}{v} ------(3)$

The group velocity of DE-BROGLIE is given by $V_g = \frac{d\omega}{dK} = \frac{d\omega/dv}{dK/dv} -----(4)$

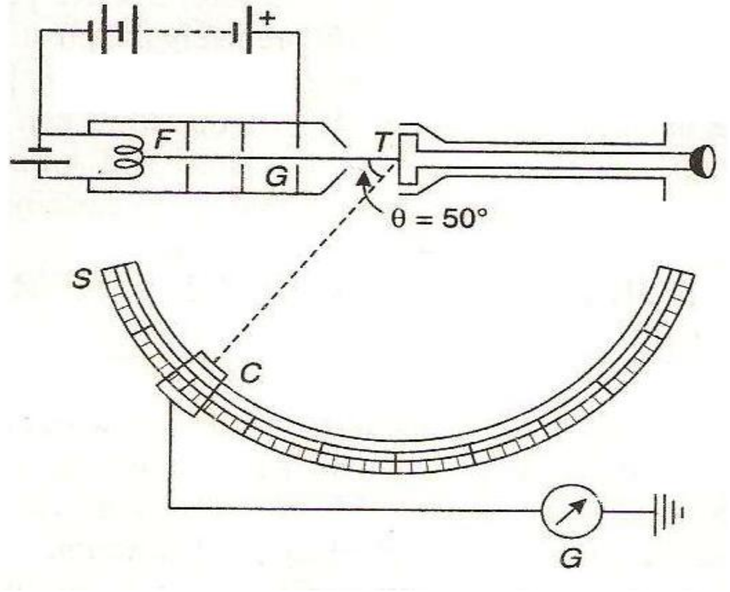
From eq. (1) we have $\frac{d\omega}{dv} = \frac{2\pi m_o v}{h(1-\frac{v^2}{c^2})^{\frac{3}{2}}} -----------------------(5)$

From eq. (2) we have $\frac{dK}{dv} = \frac{2\pi m_o}{h(1-\frac{v^2}{c^2})^{\frac{3}{2}}} ---(6) \therefore V_g = v ---------------(7)$

The DE-BROGLIE wave group associated with a moving particle travels with the same velocity as that of the particle.

ELECTRON DIFFRACTION EXPERIMENT BY DAVISSION AND GERMER:

In 1927 two American physicists, Davisson and Germer succeeded in measuring the DE-BROGLIE wavelength associated with slow electrons. The experimental arrangement is as shown in the fig.



The apparatus consists of an electron gun G where the electrons are produced and obtained in a fine pencil of electronic beam of known velocity. The electron consists of a tungsten filament F heated to dull red. Electrons are emitted from the filament due to thermionic emission. The electrons are collimated by suitable slits to obtain a fine beam. The beam of electrons is directed to fall on a large single crystal of nickel. This is known as target T. the electrons are diffracted in different directions like a wave. The angular distribution is measured by an electron detector which is connected to a galvanometer. The faraday cylinder can move on a circular scale S between 29^o to 90^o to receive the reflected electrons. A retarding potential is m reflected electrons. A retarding potential is maintained between two walls of faraday cylinder. Hence it receives only fast moving from the electron gun. The secondary electrons produced by collisions with atoms from nickel target are reflected by faraday cylinder. Hence the galvanometer deflection is only due to electrons from electron gun.

EXPERIMENTAL PROCEDURE:

Let us consider the beam of electrons is incident normally on the crystal surface. The surface layer of the crystal acts as a diffraction grating.

For each azimuth, a beam of low voltage electrons is made to fall normally on the crystal surface. The collector is moved to various positions along the circular scale S. At each position the deflection in the galvanometer is noted. This deflection is a measure of the intensity of the scattered electrons. A graph is then plotted between galvanometer current against angle θ between incident beam and the beam entering the cylinder. The experiment is repeated for several accelerating voltages V. this is shown in the fig (2)

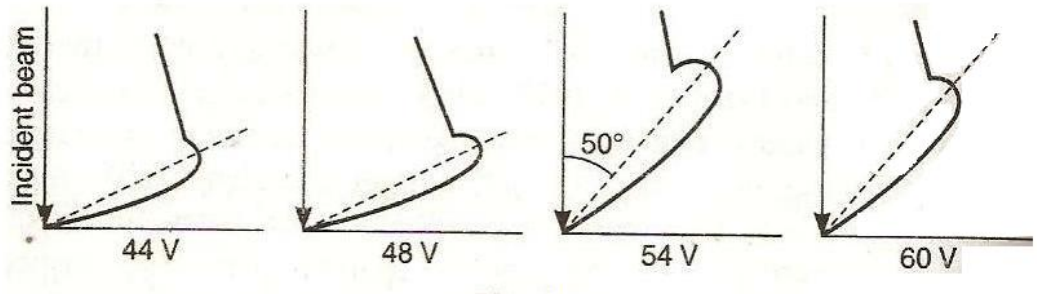
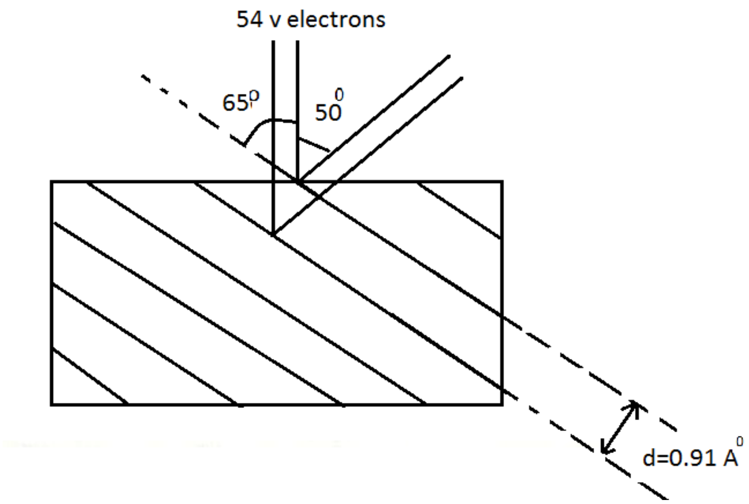


Fig. (2)

It is observed that a bump begins to appear in the curve for 44 volt. The following points are observed.

(1) If the potential is increased then the bump is moved upward.
(2) The bump becomes most prominent in the curve for 54V at $\theta = 50^0$
(3) At higher potentials the bump gradually dis-appears.

According to DE-BROGLIE wavelength associated with electrons accelerated through a potential V is given by

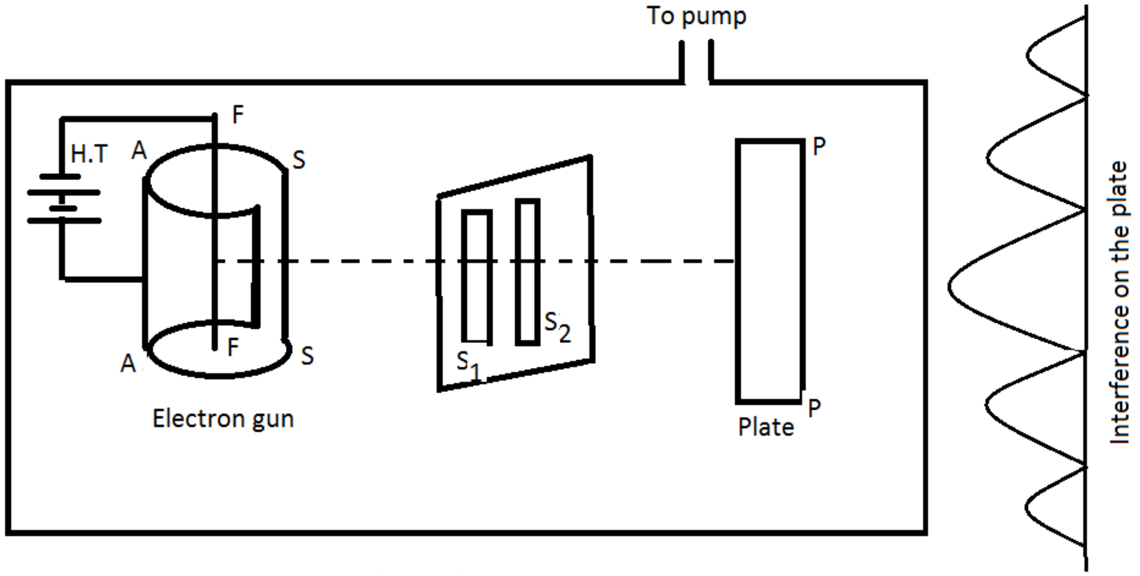


$$\lambda = \frac{12.26}{\sqrt{V}} A^0$$

Here wavelength associated with 54V electrons is $\lambda = \frac{12.26}{\sqrt{54}} = 1.67A^0$

From X-ray analysis, it is known that a nickel crystal acts as plane diffraction grating with grating space d = $0.91A^0$. According to experiment, the wavelength of the electron beam is $\lambda = 2d \sin \theta' = 2(0.91) \sin 65^\circ = 1.65 A^\circ$. This is in good agreement with the wavelength calculated from the DE-BROGLIE hypothesis. Hence confirms the DE-BROGLIE concept of matter waves.

DOUBLE SLIT EXPERIMENT:



This experiment proves the interference of electron waves just like photons. The experimental arrangement is as shown in the fig.

In the fig electron gun contains a filament FF and a cylindrical anode AA. electron gun supplies a mono energetic beam passes through a narrow slit SS. two slits S_1 and S_2 very close to each other is placed in the path of electron beam. PP is a photographic plate. The whole chamber is evacuated.

The electron beam is diffracted by slits S_1 and S_2 which acts as coherent sources. Hence the diffracted beams of electrons interfere and produce interference pattern on photo graphic plate. The interference pattern shows the existence of waves associated with electrons.

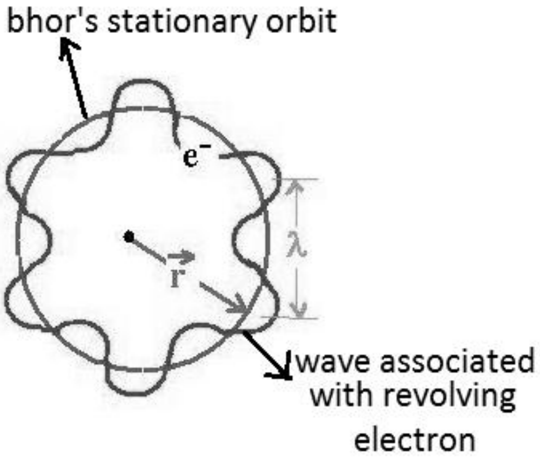
Knowing the separation between S1 and S2 and their distance from photographic plate, the wave length of the electron beam can be calculated. This is equal to the wavelength obtained by DE-BROGLIE relation. This proves the existence of matter waves associated with material particles.

STANDING DE-BROGLIE WAVES OF ELECTRON IN BOHR ORBITS:

According to DE-BROGLIE hypothesis, an electron of mass m in motion with velocity v is associated with a wave whose wavelength λ is given by $\lambda = \frac{h}{mv}$ ----- (1)

According to Bhor electron does not radiate energy while moving in an orbit. The wave associated with electron must be stationary wave and also there should be no transfer of energy.

DE-BROGLIE on the basis of his hypothesis proposed an atomic model in which stationary orbits of the Bhor's model were retained but with the difference that electron in various orbits behaves as wave. This is shown in the fig.



He suggested that stationary orbits are those in which orbital circumference ($2\pi r$) is an integral multiple of DE-BROGLIE wavelength λ. $2\pi r = n\lambda$ where n=1, 2…

But $\lambda = \frac{h}{mv}$ angular momentum of electron L=m v r =n. $\frac{h}{2\pi}$ --------- (2)

Esq. (2) is nothing but the Bhor's postulate i.e., the total angular momentum of the moving electron is an integral multiple of $\frac{h}{2\pi}$. Thus the new concept of DE-BROGLIE confirms Bhor's postulate.

UNCERTAINTY PRINCIPLE:

CONSEQUENCES OF DE-BROGLIE CONCEPT:

(1) The wave properties associated with electrons are utilized to design electron microscope. It has 1000 times better resolution than optical microscope.
(2) De-Broglie explained the quantization of angular momentum in terms of wave nature of moving electron around the nucleus.
(3) The neutron diffraction is a quite common technique for investigation of the structure of matter.
(4) De-Broglie concept of matter waves leads to Bhor's principle of complimentary which states that the two apparently different views of wave and particle.
(5)De-Broglie concept leads to un-certainty principle. The material particle exhibits particle nature as well as wave nature. But the two natures can't be exhibited simultaneously. This leads to un-certainty principle.

HEISENBERG UNCERTAINTY PRINCIPLE:

According to him "it is impossible to measure both position and momentum of a particle simultaneously to any desired degree of accuracy." Qualitatively this principle states that "the order of magnitude of the uncertainties in the knowledge of two variables must be at least Planck's constant h.

Considering the pair of physical variables as position and momentum, we have

$$\Delta p \Delta x \approx h ----------------(1)$$

where, Δp is the uncertainty in determining the momentum and Δx is the uncertainty in determining the position of the particle. Similarly we have

$$\Delta E \Delta t \approx h ------------------(2)$$

$$\Delta J \Delta \theta \approx h ------------------(3)$$

Where ΔE and Δt are un-certainties in determining the energy and time. While ΔJ and $\Delta \theta$ are uncertainties in determining angular momentum and angle.

PRINCIPLE:

The product of un-certainties in determining the energy and momentum of the particle can never be smaller than the order of $\frac{h}{4\pi}$. So we have

$$\Delta p \Delta x \geq \frac{h}{4\pi}$$

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

$$\Delta J \Delta \theta \geq \frac{h}{4\pi}$$

TIME ENERGY UNCERTAINTY PRINCIPLE:

Let us consider the case of a free particle with rest mass m_o moving along x-direction with velocity V_x.

The kinetic energy $E = \frac{1}{2}m_o v_x{}^2 = \frac{P_x{}^2}{2m_o}$ ------------(1)

If P_x and ΔE be the uncertainties in momentum and energy, then differentiating eq. (1) we get

$$\Delta E = \frac{2P_x\Delta P_x}{2m_o} \Rightarrow P_x\Delta P_x = m_o\Delta E$$

$$\therefore \Delta P_x = \frac{m_o}{P_x}\Delta E \Rightarrow \Delta P_x = \frac{1}{V_x}\Delta E \text{ --------------(2)}$$

Let the un-certainty in time interval for measurement at point x be Δt, then uncertainty Δx in position is

$$\Delta x = V_x\Delta t \text{ ------------------(3)}$$

$$\therefore V_x = \frac{\Delta x}{\Delta t}$$

By substituting the value of V_x in eq. (2) we get

$$\Delta P_x = \frac{\Delta t}{\Delta x}\Delta E \Rightarrow \Delta x\Delta P_x = \Delta t\Delta E \text{ --------(4)}$$

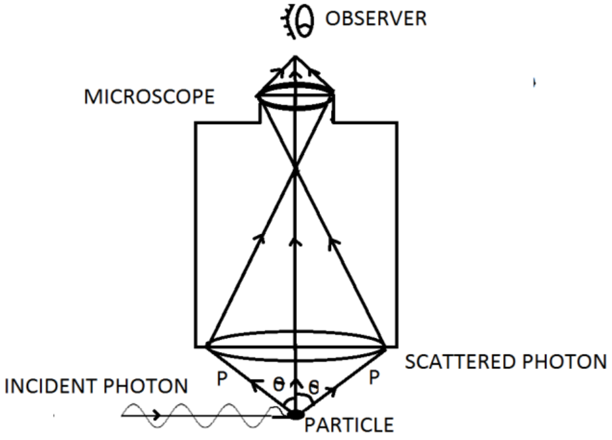
We know that $\Delta x\Delta P_x \geq \frac{h}{4\pi}$ $\quad \therefore \Delta t\Delta E \geq \frac{h}{4\pi}$ ----------------(5)

Hence the product of un-certainty in energy and un-certainty in time can never be smaller than $\frac{h}{4\pi}$.

From esq. (5) it is clear that the system that remains in a certain state for a very long time can have a very well defined energy. But shorter the time of stay, larger will be uncertainty in corresponding energy. An atom can live in the exited state only for about 10^{-8} s. Thus the concept of Bhor's orbits violates the un-certainty principle.

CONSEQUENCES OF UN-CERTAINTY RELATION:

(1) DETERMINATION OF THE POSITION OF A PARTICLE BY GAMMA RAY MICROSCOPE:



Let us measure the position of the particle in the field of gamma ray microscope. The least distance between two close points that can be resolved by the microscope is given by

$$\Delta x \approx \frac{\lambda}{2\sin\theta} -------------(1)$$

Where λ is the wavelength of the light used, and θ is the semi-vertical angle of the cone of the light and Δx is the un-certainty in determining the position of the particle.

In order to observe the electron, it is necessary that at least one photon must strike the electron and scatter inside the microscope. When the photon of initial momentum P=$\frac{h}{\lambda}$. After scattering enters the field of view of microscope, it may be anywhere with an angle 2θ. Thus its x component of momentum P_x may lie between $P\sin\theta$ and $-P\sin\theta$. Since the momentum is conserved during the collision, the un-certainty in the measurement of the x component of the momentum is

$$\Delta P_x = P\sin\theta - (-P\sin\theta) = 2p\sin\theta -------------(2)$$

$$\Delta P_x = \frac{2h}{\lambda}\sin\theta$$

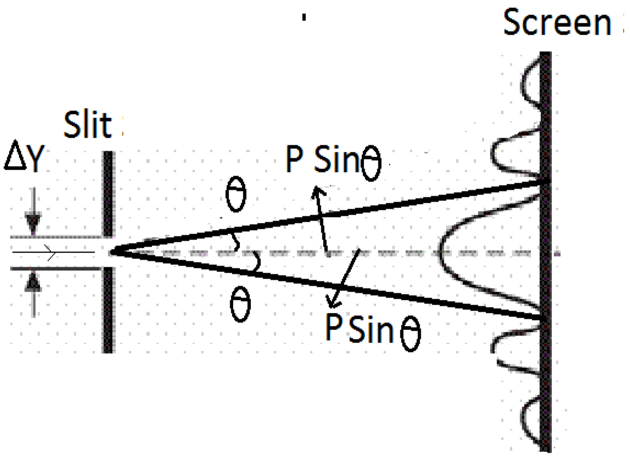
From eq. (1) and eq. (2) we have

$$\Delta x \Delta P_x \approx \frac{\lambda}{2\sin\theta}.\frac{2h}{\lambda}\sin\theta$$

$$\Delta x \Delta P_x \approx h ----------------(3)$$

From the above equation it is clear that the product of un-certainties in position and momentum is of the order of Planck's constant.

(2) DIFFRACTION BY A SINGLE SLIT:



As shown in the fig, suppose a narrow beam of electron passes through the single narrow slit and produces a diffraction pattern on the screen. The first minimum of the pattern is obtained by putting n=1 in the equation $d\sin\theta = n\lambda$

$$\therefore \Delta y \sin\theta = \lambda -------------(1)$$

Here Δy is the width of the slit and θ is the angle of deviation corresponding to first minimum.

In producing diffraction pattern on the screen all the electrons have passed through the slit. But we cannot say definitely at what place of the slit. Hence the un-certainty in determining the position is equal to the width Δy of the slit.

$$\therefore \Delta y = \frac{\lambda}{2\sin\theta} ---------------(2)$$

After diffraction at the slit the electrons deviated from their initial path. Hence un-certainty in Y-component of momentum is

$$\Delta P_y = 2P\sin\theta = 2\frac{h}{\lambda}\sin\theta --------------(3)$$

(2) POSITION OF ELECTRON IN BOHR ORBIT:

In calculating the radius of Bhor's first orbit: If Δx *and* ΔP are the uncertainties in position and momentum of the electron in the first orbit, then according to uncertainty principle we have,

$$\Delta x.\Delta P \geq \frac{h}{4\pi}$$

$$\Delta P \geq \frac{h}{4\pi\Delta x} ----------------(1)$$

The uncertainty in K.E. , ΔT of the electron may be written as

$$\Delta T = \frac{1}{2}m(\Delta v)^2 \geq \frac{m^2(\Delta v)^2}{2m} \geq \frac{(\Delta P)^2}{2m}$$

$$\Delta T \geq \frac{h^2}{16\pi^2 2m(\Delta x)^2} -----------------(2)$$

The un-certainty in the potential energy ΔV of the same of the electron is given by

$$\Delta V \geq \frac{-2e^2}{\Delta x} -------------------(3)$$

The un-certainty in total energy ΔE is given by

$$\Delta E \geq \Delta T + \Delta V \geq \frac{h^2}{16\pi^2 2m(\Delta x)^2} + \frac{-2e^2}{\Delta x} ---------------(4)$$

The un-certainty in the energy will be minimum if

$$\frac{d(\Delta E)}{d(\Delta x)} = 0 \text{ and } \frac{d^2(\Delta E)}{d^2(\Delta x)} \text{ is positive.}$$

Now $\frac{d(\Delta E)}{d(\Delta x)} \geq \frac{-2h^2}{16\pi^2 2m(\Delta x)^3} + \frac{2e^2}{(\Delta x)^2} = 0$

$$\frac{h^2}{16\pi^2 m(\Delta x)^3} \approx \frac{2e^2}{(\Delta x)^2} \Rightarrow \Delta x \approx \frac{h^2}{16\pi^2 m 2e^2} ----------------(5)$$

From esq. (2) and eq (3) we have

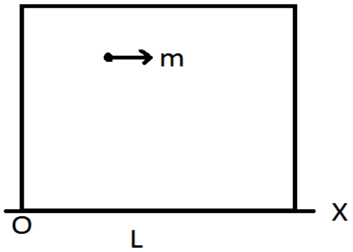
$$\Delta y.\Delta P_y \approx \frac{\lambda}{2\sin\theta}.\frac{2h\sin\theta}{\lambda} \approx h$$

$$\Delta y.\Delta P_y \approx h -------------------(6)$$

This relation shows that the product of un-certainties in position and momentum is of the order of Planck's constant.

CONSEQUENCES OF UN-CERTAINTY RELATION:

(i) Particle in the box: consider a particle of mass m moving in one-dimensional box of length L in X-direction as shown in the fig. the minimum un-certainty in the position x of the particle will be $\Delta x = L$. Hence L is the width of the box.



According to un-certainty principle $\Delta x.\Delta P \approx \hbar \Rightarrow \Delta P \approx \frac{\hbar}{\Delta x} \approx \frac{\hbar}{L}$

Approximately, the above equation can be written as $P \approx \Delta P \approx \frac{h}{2L}$

Energy E= $\frac{P^2}{2m} \approx \frac{h^2}{8mL^2}$ $\quad E \approx \frac{\pi^2\hbar^2}{2mL^2}$

If we calculate the value of $\frac{d^2(\Delta E)}{d(\Delta x)^2}$ it becomes positive and hence Δx given by the equation (5) is minimum. This shows that the energy of the electron is minimum in the first orbit. Hence the radius of the first orbit is given by

$$r \approx \Delta x \approx \frac{h^2}{16\pi^2 mze^2}$$

$$r \approx \frac{\hbar^2}{4mze^2}$$

Where $\hbar = \frac{h}{2\pi}$

COMPLEMENYTARY PRINCIPLE OF BOHR:

The most important consequences of un-certainty principle are it is impossible to determine the wave and particle properties at the same time in a single experiment.

According to complementary principle, the wave and the particle nature of matter and light complementary rather than contradictory.

Let us consider an experimental arrangement in which light is diffracted by a double slit and is detected on a screen that consists of many adjacent photo-electric cells. The photo electric cells respond to photons. If we plot the no: of photons, against the location of cell, a wave like pattern is obtained. It follows that any experiment which can be devised displays either the particle like characteristics or wave-like characteristics of the system. The wave and the particle pictures give complementary description of the same system.

QUANTUM (WAVE) MECHANICS

The classical mechanics fails to explain the microscopic system of particles due to uncertainty principle. According to uncertainty principle, the position and momentum of a particle can't be measured accurately at the same time. The measurement of one quantity introduces an uncertainty into the other. According to de-broglie, a material particle is associated with a wave. So, a mathematical reformation using a wave function associated with matter waves is needed. So, quantum mechanics was developed in 1926 by schroedinger.

Basic postulates of quantum mechanics :

SCHROEDINGER WAVE EQUATION:

(1) TIME INDEPENDENT WAVE EQUATION:

According to de-broglie theory, a particle of mass m is always associated with a wave whose wavelength is given by $\lambda = \frac{h}{mv}$ .Consider a system of stationary waves associated with a particle. Let x, y, z be the coordinates of the particle and ψ be the displacement for the de-broglie at any time t. ψ is called wave function. Here ψ is finite, single valued and periodic function.

The classical differential equation of a wave motion is given by

$$\frac{\partial^2\psi}{\partial t^2} = v^2\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) = v^2\nabla^2\psi \text{-------------}(1)$$

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ here ∇^2 is the laplacian operator

v=wave velocity

The solution of eq (1) is given by

$$\psi = \psi_0 \sin\omega t = \psi_0 \sin 2\pi\upsilon t \text{-----------}(2)$$

Where υ=frequency of stationary wave

Differentiating the above equation we get

$$\frac{\partial\psi}{\partial t} = \psi_0 2\pi\upsilon \cos 2\pi\upsilon t$$

$$\frac{\partial^2\psi}{\partial t^2} = -\psi_0(2\pi\upsilon)^2 \sin 2\pi\upsilon t$$

$$\frac{\partial^2\psi}{\partial t^2} = -4\pi^2\upsilon^2\psi = -\frac{4\pi^2 v^2}{\lambda^2}\psi \text{------------}(3)$$

By substituting the above value in eq (1) we get

$$v^2\nabla^2\psi = -\frac{4\pi^2 v^2}{\lambda^2}\psi$$

$$\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0$$

$$\nabla^2\psi + \frac{4\pi^2}{h^2}m^2v^2\psi = 0 \text{------------------}(4)$$

If E and V be the total and potential energies of the particle respectively the K.E is given by

$$\frac{1}{2}mv^2 = E - V$$

$$\frac{m^2v^2}{2m} = E - V \Rightarrow m^2v^2 = 2m(E-V) ----------(5)$$

From eq (4) and eq (5) we have

$$\nabla^2\psi + \frac{4\pi^2}{h^2}2m(E-V)\psi = 0 ----------(6)$$

This is the schroedinger time dependent wave equation

Eq (6) can be written as

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

$$\frac{\hbar^2}{2m}\nabla^2\psi + (E-V)\psi = 0$$

$$\frac{\hbar^2}{2m}\nabla^2\psi - V\psi = -E\psi$$

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi = E\psi$$

$$\hat{H}\psi = E\psi$$

Here $\hat{H} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]$

And H is known as Hamiltonian operator

For a free particle v=0

$$\nabla^2\psi + \frac{2mE}{\hbar^2}\psi = 0 ------------(7)$$

(2) SCHROEDINGER TIME DEPENDENT WAVE EQUATION:

In order to derive time dependent wave eq, schroedinger introduced a mathematical function ψ which is variable quantity associated with the moving particle. This is a complex function of space coordinates of the particle and time.

The differential equation representing a one dimensional wave motion is

$$\frac{\partial^2\psi}{\partial t^2} = v^2\nabla^2\psi ----------------(1)$$

The solution of the above equation with respect to time t, we get

$$\psi(x,y,z,t) = \psi_0(x,y,z)e^{-i\omega t} ------------(2)$$

Differentiating the above equation w.r.t to time t we get

$$\frac{\partial\psi}{\partial t} = \psi_0(-i\omega)e^{-i\omega t}$$

$$= \psi_0(-i2\pi v)e^{-i\omega t} = -2\pi i v\psi = -2\pi i\left(\frac{E}{h}\right)\psi = \frac{-iE}{\hbar}\psi$$

$$E\psi = i\hbar\frac{\partial\psi}{\partial t} ------------------(3)$$

Substituting the above value in schroedinger time independent wave eq we get

$$\nabla^2\psi+\frac{2m}{\hbar^2}\left[i\hbar\frac{\partial\psi}{\partial t}-V\psi\right]=0$$

$$\nabla^2\psi=\frac{-2m}{\hbar^2}\left[i\hbar\frac{\partial\psi}{\partial t}-V\psi\right]$$

$$\frac{-\hbar^2}{2m}\nabla^2\psi+V\psi=i\hbar\frac{\partial\psi}{\partial t}----------------(4)$$

This eq is known as schroedinger time dependent wave equation.
Eq (4) can be written as

$$[\frac{-\hbar^2}{2m}\nabla^2+V]\psi=i\hbar\frac{\partial\psi}{\partial t}----------------(5)$$ Here $\hat{E}=i\hbar\frac{\partial}{\partial t}$ =energy operator

$or\ \ \hat{H}\psi=\hat{E}\psi$

Eq (5) describes the motion of a non-relativistic material particle.

PHYSICAL SIGNIFICANCE OF WAVE FUNCTION :

The interpretation of ψ was given by schroedinger himself in terms of charge density. If A is the amplitude of electromagnetic wave then the energy density is equal to A^2 .Hence photon density is equal to $\frac{A^2}{h\upsilon}$ or photon density is proportional to A^2. If ψ is the amplitude of matter waves at any point in space, then the particle density at that point is proportional to ψ^2 therefore ψ^2 is a measure of particle density. When this is multiplied by the charge of the particle, the charge density is obtained.

It is observed that in some cases, ψ is appreciably different from zero with in a wave packet. To explain it, max born suggested a new idea about physical significance of ψ .According to Max born $\psi\psi^*=|\psi|^2$ gives the probability of finding the particle in the state ψ . ψ^2 is a measure of probability density. The probability of finding the particle in volume $d\tau=dxdydz$ is given by $|\psi|^2 dxdydz$. For the total probability of finding the particle somewhere in space is unity.

$$\iiint|\psi|^2 dxdydz=1$$

ψ Satisfying the above requirement is said to be normalized.

Wave function properties:

1) IT MUST BE FINITE EVERYWHERE:

If ψ is infinite at a particular point, then it would mean an infinitely large probability of finding the particle at that point.

This is not possible hence ψ it must have a finite or zero value at any point.

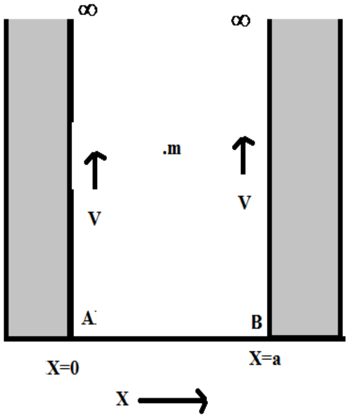
2) IT MUST BE SINGLE VALUED:

Let us consider that ψ has more than one value at any point. It means that there is more than one value of probability of finding the particle at that point.

3) IT MUST BE CONTINUOUS:

For Schrödinger equation $\frac{d^2\psi}{dx^2}$

Must be finite everywhere. This is possible only when $\frac{d\psi}{dx}$ has no discontinuity at any boundary where potential changes. Hence ψ too must be continuously across a boundary.



<u>Application of Schrodinger wave equation to particle in one dimensional infinite box:</u>

Let us consider the case of a particle of mass m moving along the x-axis between two rigid walls A and B at x=0 and x=∞. The particle is free to move between the walls. The potential energy of the particle between the two walls is constant because no force is acting on the particle. The constant potential energy is taken as zero for simplicity. When the particle strikes any one of the walls, it is reflected back immediately .Now the force acting on the particle abruptly changes from zero to a finite value F within a distance of zero at the wall. The potential function is defined in the following way

$v(x)=\infty$ For x<0 and x>a

$v(x)=0$ For $0 \le x \le a$

Schrödinger wave equation for the particle is given by

$$\frac{d^2\psi}{dx^2}+\frac{8\pi^2 m}{h^2}(E-V)=0$$

As V=0 between the walls, hence the eqn. becomes

$$\frac{d^2\psi}{dx^2}+\frac{8\pi^2 m}{h^2}E\psi=0---------(1)$$

Let $\frac{8\pi^2 m}{\hbar^2}E=K^2$ then the above eq. becomes

$$\frac{d^2\psi}{dx^2}+K^2\psi=0-------------(2)$$

The general solution of the above eqn. is given by

$\psi(x)=A\sin kx+B\cos kx$

Where A and B are constants

By applying boundary conditions we can determine the value of constants A and B. The particle can’t penetrate the walls, hence ψ =0 at x=0 and ψ =0 at x=a these are the boundary conditions. Hence

0=Asin0+Bcos0 i.e. B=0

$\psi(x)=A\sin kx$

Again 0=A sin ka $\because \psi(x)=0$ when x=a

Here A$\neq 0$ hence ka=0

sin ka=0 or ka=nπ n=0,1,2

$$k = \frac{n\pi}{a} \text{--------}(3)$$

Now the wave function becomes

$$\psi(x) = A\sin\frac{n\pi x}{a} \text{--------}(4)$$

<u>EIGEN VALUES:</u> From eqn. (3) we have

$$k^2 = \frac{n^2\pi^2}{a^2} \text{--------}(5)$$

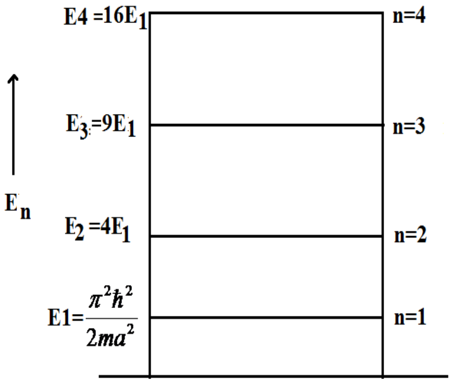
And also $$k^2 = \frac{8\pi^2 mE}{h^2} \text{--------------}(6)$$

From eq (5) and eq (6) we have

$$\frac{n^2\pi^2}{a^2} = \frac{8\pi^2 mE_n}{h^2}$$

$$E_n = \frac{n^2h^2}{8ma^2} = n^2\frac{\pi^2\hbar^2}{2ma^2} \text{-------------}(7)$$

From esq. (7) it is clear that inside an infinitely deep potential wall, the particle can have only discrete set of values of energy. Hence the energy of the particle is quantized.



If n=1 then $E_1 = \frac{\pi^2\hbar^2}{2ma^2}$

n=2 then $E_2 = \frac{4\pi^2\hbar^2}{2ma^2} = 4E_1$

n=3 then $E_3 = \frac{9\pi^2\hbar^2}{2ma^2} = 9E_1$

n=4 then $E_4 = \frac{16\pi^2\hbar^2}{2ma^2} = 16E_1$

The energy level is as shown in the fig (2)

<u>WAVE FUNCTION:</u> by applying normalization condition we get

fig (2)

$$\int_0^a |\psi(x)|^2 dx = 1$$

$$\int_0^a A^2 \sin^2\frac{n\pi x}{a} dx = 1$$

$$A^2\int_0^a \frac{1}{2}\left(1-\cos\frac{2n\pi x}{a}\right)dx = 1$$

$$\frac{A^2}{2}\left[x - \frac{a}{2\pi n}\sin\frac{2n\pi x}{a}\right]_0^a = 1$$

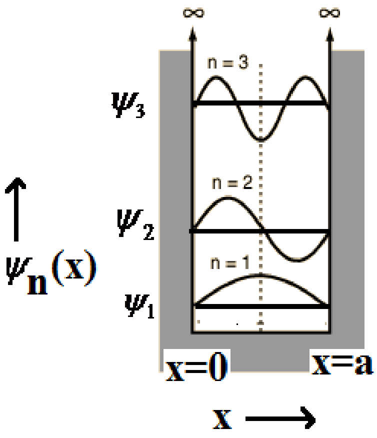


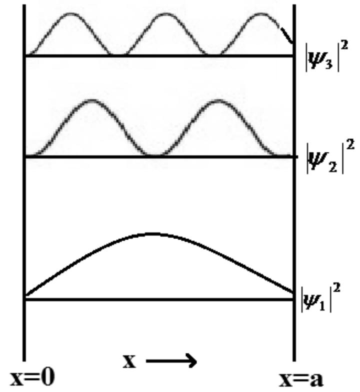
fig (3)

$\frac{A^2 a}{2}=1$ Or $A^2=\frac{2}{a}$ or $A=\sqrt{\frac{2}{a}}$

$$\psi(x)=\sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}\text{---------------(8)}$$

The above Eqn. represents, the wave function of the particle enclosed in infinitely deep potential well. The wave functions ψ_1,ψ_2 and ψ_3 corresponding to n=1,2and3 are shown in fig (3)

PROBABILITY DENSITY: The probability of finding the particle between portions x and x+dx is given by $P(x)=|\psi|^2 dx=\frac{2}{a}\sin\frac{n\pi x}{a}dx$



Probability density $P(x)=\frac{2}{a}\sin\frac{n\pi x}{a}$

It is maximum when $\frac{n\pi x}{a}=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2}$...............

Or $x=\frac{a}{2n},\frac{3a}{2n},\frac{5a}{2n}$..............

For n=1 the particle is most likely to be found at the middle of the box because $|\psi_1|^2$ is maximum at x=a/2 for n=2, x=a/4 and 3a/4 i.e. the particle is most likely too be found at x=a/4 and 3a/4 because $|\psi_2|^2$is maximum at that points. Similarly, for n=3 the particle is most likely to be found at x=a/6,3a/6,5a/6.the variation of probability densities $|\psi_1|^2$, $|\psi_2|^2$and$|\psi_3|^2$. For n=1, 2 and 3 are shown in the fig.

According to classical mechanics, a particle moving with a velocity v spends a time $\frac{dx}{v}$ in traveling distance dx and hence the probability distribution is constant. According to Quantum mechanics the probability of the particle being present is different for different quantum numbers.

Quantum mechanics results vary drastically from classical results. The classical mechanics predicts the same probability for the particle being any where in the box while quantum mechanics predicts that the probability is different at different points.

RADIOACTIVITY DECAY:

{SYLLABUS:Alpha decay: basics of α-decay processes. Theory of α-decay, Gamow's theory, Geiger Nuttal law.β-decay, Energy kinematics for β-decay, positron emission, electron capture, neutrino hypothesis. }

Basics of α-decay processes: **α-decay** is a type of radioactive decay in which an atomic nucleus emits an alpha particle and thereby 'decays' into a different atomic nucleus, with a mass number that is reduced by four and an atomic number that is reduced by two. It has a charge of +2 *e* and a mass of 4 u. For example, uranium-238 decays to form thorium-234.

$$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$$

Alpha decay typically occurs in the heaviest nuclides. Theoretically, it can occur only in nuclei somewhat heavier than nickel , where the overall binding energy per nucleon is no longer a minimum and the nuclides are therefore unstable toward spontaneous fission-type processes. Alpha particles have a typical kinetic energy of 5 MeV and have a speed of about 1.5 x 10^{-7}m/s.

RANGE OF α PARTICLES: The distance through which an α particle travels in a substance before coming to rest is called the range of α particle in that substance.

Theory of α-decay :

GEIGER-NUTTAL LAW:

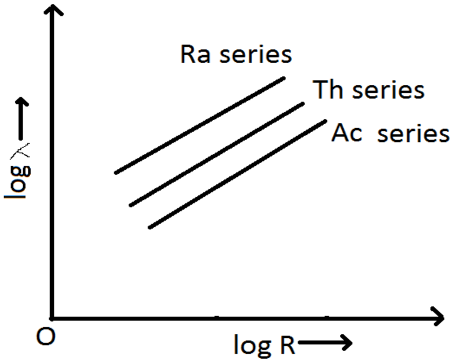
We know that different α emitters emit α particles of different energies and hence of different ranges. Higher energy α particles have the shortest half-life period (largest decay constant) and vice-versa.

Geiger and Nuttal found that these exists a regular relationship between the ranges and the half-lives of the element. The relationship is expressed as

$$\log\lambda = A + B\log R$$

The above relation is known as Geiger-Nuttal law. The law is helpful in determining roughly the decay constants of radio-active substances of very short or very long lives.

If a graph is plotted between $\log\lambda$ and log R for various radio-active substances, the graph is as shown in the fig. The three lines have different intercepts on log R axis. This indicates that A is different for different series while B is same for all series.

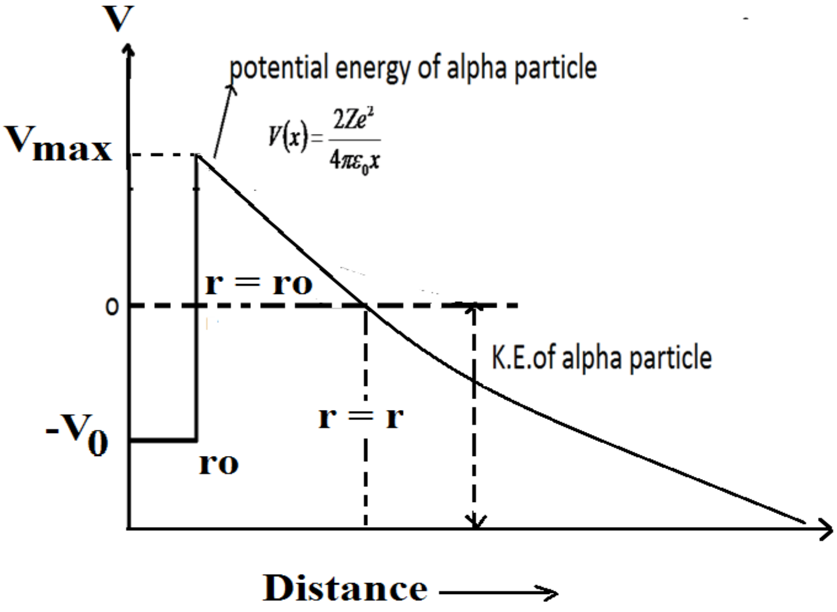


When the dis-integration constant is high, the range is also high.

α -DECAY: In order to escape α particle from the nucleus it must have sufficient kinetic energy .Only the α particle mass is sufficiently smaller than sum of masses of its constituent nucleons. Therefore in the formation of α particle within the nucleus sufficient energy is released.

If α particle approaches the nucleus from infinity, work has to be done on it against coulomb's repulsive force. This is equal to potential energy of the system.

Due to attractive forces within the nucleus ,the α particle has negative potential



energy(-V_o).the region of potential negative potential from r_o to 0 is called the potential well.).the region of potential positive potential is called as the potential barrier. The maximum potential energy V_{max} is known as barrier height.

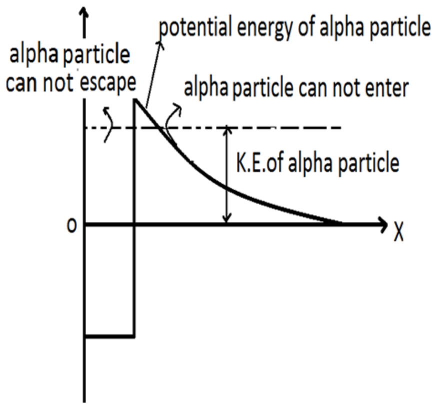
If α particle is emitted from uranium nucleus, it would have energy equal to or greater than 27MeV.But α particles emitted by uranium nucleus, have energy of only 4MeV.Then how they get out across the barrier.

GAMOW'S THEORY OF α-DECAY:

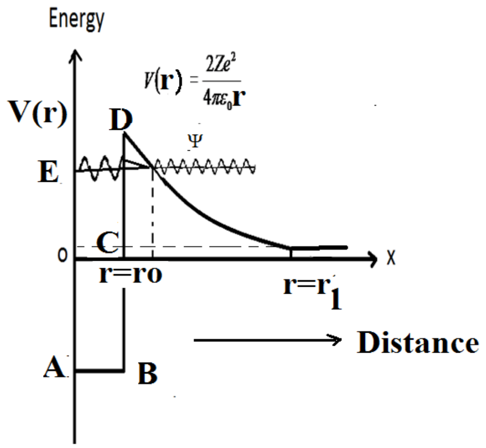
According to classical mechanics, α particle cannot escape from the nucleus .Hence classical mechanics fails to explain α decay. Quantum mechanics successfully explains how an α particle to leak through the potential barrier even though its K.E is less than the P.E of the height of the potential barrier. This probability of leaking of an α particle through the barrier is known as Tunnel effect.

GAMOW MADE THE FOLLOWING ASSUMPTIONS:

(1) The α particle is pre-formed inside the parent nucleus.
(2) The α particle is in constant motion and bounces back and forth from the barrier walls.
(3) In each collision with the wall of the potential barrier, there is a definite probability of leakage through the barrier.



According to classical mechanics



According to quantum mechanics

Let υ be the frequency of α-particle collides with the walls in order to escape from the nucleus and P be the probability of transmission in each collision. Then the decay probability per unit time λ is given by

$$\lambda = \upsilon P ----(1)$$

If there is only one α-particle in the nucleus which moves back and forth along the nuclear diameter then $\upsilon = \frac{V}{2r_0}$ where r_0=nuclear radius

V=velocity of αparticles

$$\therefore \lambda = \left(\frac{V}{2r_0}\right)P --------(2) \qquad \because \upsilon = \frac{V}{2r_0}$$

According to classical mechanics P=0. In quantum mechanics a moving particle is regarded as a wave. Due to tunnel effect the result is small but definite value for P. the probability P can be calculated quantum mechanically using W.K.B(**Wentzel, Kramers, Brillouin)** perturbation theory. According to this theory

$$\log_e P = \frac{-2}{h/2\pi}\int_{r_0}^{r_1}\sqrt{2m(V(r)-E)}dr ---------(3)$$ where m=mass of α particle

Further $V(r) = \frac{2Ze^2}{4\pi\varepsilon_0 r}$

V(r) is electrostatic P.E of an α particle at a distance r from the centre of nucleus of charge Ze. The region from $r=r_0$ and $r=r_1$ is known as thickness of the potential barrier.

$$\log_e P = \frac{-2}{\frac{h}{2\pi}}\int_{r_o}^{r_1}\sqrt{2m(\frac{2Ze^2}{4\pi\varepsilon_0 r}-E)}dr -------(4)$$

When $r=r_1$, E=V $\quad \therefore E = \frac{2Ze^2}{4\pi\varepsilon_0 r_1}$

$$\log_e P = \frac{-2}{\frac{h}{2\pi}}\int_{r_o}^{r_1}\sqrt{2m(\frac{ER_1}{r}-E)}dr$$

$$\log_e P = \frac{-2}{\left(\frac{h}{2\pi}\right)}\left(2mE\right)^{\frac{1}{2}}\int_{r_o}^{r_1}\left(\frac{r_1}{r}-1\right)^{\frac{1}{2}}dr$$

Substitute r= $r_1\cos^2\theta$ and r$_o$= $r_1\cos^2\theta_0$ and on simplification we get

$$\log_e P = \frac{-2}{\left(\frac{h}{2\pi}\right)}\left(2mE\right)^{\frac{1}{2}}r_1\left[\cos^{-1}\left(\frac{r_o}{r_1}\right)^{\frac{1}{2}} - \left(\frac{r_o}{r_1}\right)^{\frac{1}{2}}\left(1-\frac{r_o}{r_1}\right)^{\frac{1}{2}}\right]\text{----------------(5)}$$

If the width of the potential barrier is very large compared with the nuclear radius i.e., $r_1 >> r_0$ then

$$\cos^{-1}\left(\frac{r_o}{r_1}\right)^{\frac{1}{2}} \approx \frac{\pi}{2}-\left(\frac{r_o}{r_1}\right)^{\frac{1}{2}} \text{ And } \left(1-\frac{r_o}{r_1}\right)^{\frac{1}{2}} \approx 1$$

$$\therefore \log_e P = \frac{-2}{\left(\frac{h}{2\pi}\right)}\left(2mE\right)^{\frac{1}{2}}r_1\left[\frac{\pi}{2}-\left(2\frac{r_o}{r_1}\right)^{\frac{1}{2}}\right]\text{----------------(5)}$$

On substituting $r_1 = \frac{2Ze^2}{4\pi\varepsilon_0 E}$ we get

$$\log_e P = \frac{4e}{\left(\frac{h}{2\pi}\right)}\left(\frac{m}{\pi\varepsilon_0}\right)^{\frac{1}{2}}Z^{\frac{1}{2}}r_0^{\frac{1}{2}} - \frac{e^2}{\left(\frac{h}{2\pi}\right)\varepsilon_0}\left(\frac{m}{2}\right)^{\frac{1}{2}}ZE^{\frac{-1}{2}}$$

$$\log_e P = 2.97Z^{\frac{1}{2}}R^{\frac{1}{2}} - 3.95ZT^{\frac{-1}{2}}\text{---------------------(6)}$$

Where r is in Fermi and kinetic energy E is in MeV

From esq. (1) $\lambda = \upsilon P \Rightarrow \log\lambda = \log_e\upsilon + \log_e P \Rightarrow \log_e\left(\frac{V}{2r_o}\right) + \log_e P$.

$$\log_e\lambda = \log_e\left(\frac{V}{2R}\right) + 2.97Z^{\frac{1}{2}}r_o^{\frac{1}{2}} - 3.95ZE^{\frac{-1}{2}}\text{-----------------(7)}$$

On changing the base of log from e to 10

$$\log_{10}\lambda = \log_{10}\left(\frac{V}{2r}\right) + 0.4343\left(2.972Z^{\frac{1}{2}}R^{\frac{1}{2}} - 3.95ZE^{\frac{-1}{2}}\right)$$

$$\log_{10}\lambda = \log_{10}\left(\frac{V}{2r_o}\right) + 1.29Z^{\frac{1}{2}}r_o^{\frac{1}{2}} - 1.72ZE^{\frac{-1}{2}} \text{---------------------} (8)$$

Beta decay:

The nuclei do not contain electrons .But $\beta - decay$ is the process of emission of electrons from the nuclei. How it is possible? The answer is neutron is converted into proton as shown in the following eq.

$$n \rightarrow P + \pi^{-} \rightarrow P + e^{-} + \upsilon^{-}$$

In the case of $\beta - decay$ the mass number remain unchanged but atomic number increases by unity.

EMISSION OF POSITRON: P→n+ $\pi^{+} \rightarrow n + e^{+} + \upsilon$

EMISSION OF POSITIVE $\beta - decay$: ${}_{Z}X^{A} \rightarrow {}_{Z-1}Y^{A} + {}_{+1}\beta^{O} + \upsilon$

NEUTRINO HYPOTHESIS OF $\beta - decay$:

IN 1930 Pauli forwarded a hypothesis called neutrino hypothesis. According to this hypothesis, in the process of $\beta - decay$, a neutral particle which has negligible mass and spin half is emitted along with β-particles. This particle is known as neutrino. It carries a part of available energy and momentum. Neutrino travels with the velocity of light. On the basis of this theory we can explain conservation of linear and angular momentum in $\beta - decay$.

$\beta - decay$ can be represented by the following equation

Neutron → proton + electron + neutrino.

$${}_{0}n^{1} \rightarrow {}_{1}H^{1} + {}_{-1}e^{0} + \upsilon$$

Mass $1 \rightarrow 1 + 0 + 0$

Charge $0 \rightarrow +1 + -1 + 0$

The neutrino theory successfully explains the continuous energy spectrum of β-rays. In $\beta - decay$ the proton remains in the nucleus while the β-particles and neutrino come out. The total energy of these two particles is constant and is equal to end point of energy. The energy is shared between β-particles and neutrino.

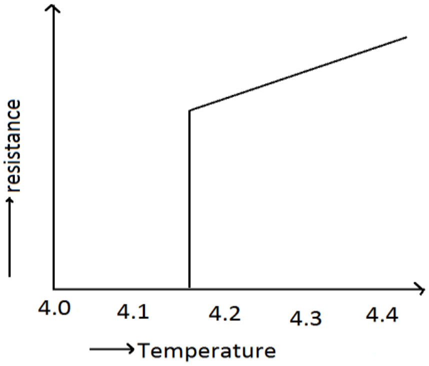
When neutrino gets no energy, β-particles are emitted with maximum energy. The energy carried by neutrino is varies from zero to maximum. Hence the spectrum is continuous.

We know that the angular momentum of the fundamental particles electron, proton and neutron is equal to $\frac{1}{2}\hbar$. When β-particle is emitted by the nucleus, the angular momentum of β -particle($\frac{1}{2}\hbar$)is removed by the angular momentum of neutrino($-\frac{1}{2}\hbar$).

SUPER CONDUCTIVITY

INTRODUCTION:

It was discovered by Kamerling onnes in 1911, when he was measuring the resistivity of mercury at low temperatures. The electrical resistance of pure mercury drops suddenly to zero at 4.15K



TRANSISTION TEMPERATURE:

The temperature at which the resistance dis-appears is called the transition temperature or critical temperature.

SUPER CONDUCTIVITY:

When the electrical resistance of a substance drops suddenly to zero, when the specimen is cooled below a certain temperature is known as super conductivity.

BASIC EXPERIMENTAL FACTS:

The following are the experimental facts of superconductivity materials

(1) At room temperature, they have high resistivity.
(2) The critical temperature is different for different isotopes of an element.
(3) T_c decreases with increasing atomic weight of isotopes
(4) By adding impurities the superconductivity of a superconducting element is not lost but its transition temperature is lowered.
(5) From X-ray diffraction studies it is clear that there is no change in the crystal structure. Hence super conductivity may be due to electrons.
(6) Due to transition the thermal expansion and elastic property do not change.
(7) All thermo electric effects dis-appear in superconductors.
(8) The presence of a magnetic field causes a critical temperature T_c of the superconductor to decrease.
(9) When a strong magnetic field above a critical field B_c is applied to a superconductor below its transition temperature, the superconducting property is destroyed.
(10)Elements like copper and silver that are good conductors' do not become superconductors when cooled.
(11) Superconductor is perfectly diamagnetic. No magnetic field can exist inside a superconductor.
(12) It can conduct electric current even in the absence of an applied voltage and the current can persist for years without any detectable decay.

EXPLANATION OF SUPERCONDUCTIVITYAND ZERO RESISTANCE:

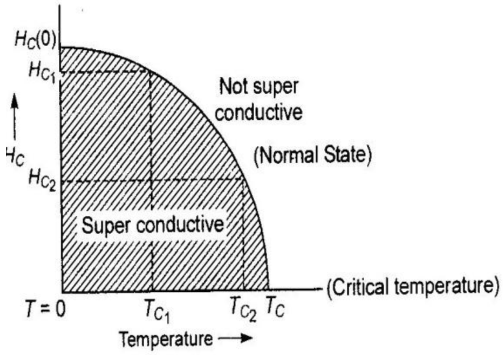
The electrical resistance of an ordinary metal is due to the collision between conduction electrons and ions of the crystal. In a superconductor, the electrons are scattered in pairs due to which an exchange force between electrons exists. This force is just similar to forces between the nucleons in a nucleus. When the electron has opposite spins and momenta, there is very strong attractive force between them. In superconducting state, the force of attraction is greater than the electrostatic forces of repulsion. Thus all conduction electrons become a bound system. Now no transfer of energy takes place from this system to lattice ions. When an electric field is applied to a substance in superconducting state, the pairs of electrons gain additional K.E. thus they give rise to a current. Since these electrons do not transfer energy to the lattice they are not transfer energy to the lattice they are not slowed down. It means that the substance does not posses any electrical resistivity. It means that the resistance of superconductor is zero.

EFFECT OF EXTERNAL FIELD:

In 1913, Kammerlingh onnes observed that, the superconducting material restores its normal resistance when a strong magnetic field is applied.

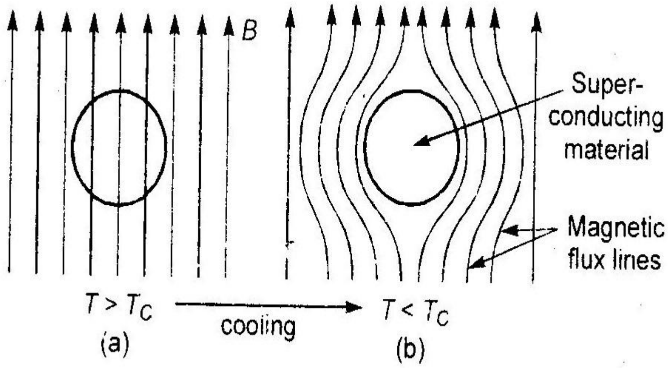
The minimum value of applied magnetic field when the superconductor loses its superconductivity is known as critical magnetic field H_c (o). If the applied magnetic field exceeds the critical value H_c (o), the superconducting state is destroyed. This is shown in the following figure.

From the fig it is clear that the normal state of the material is restored if the magnetic field is greater than the critical value or the temperature is raised above critical temperature T_c.



Showing critical magnetic field as a function of temperature

MEISSNER EFFECT:



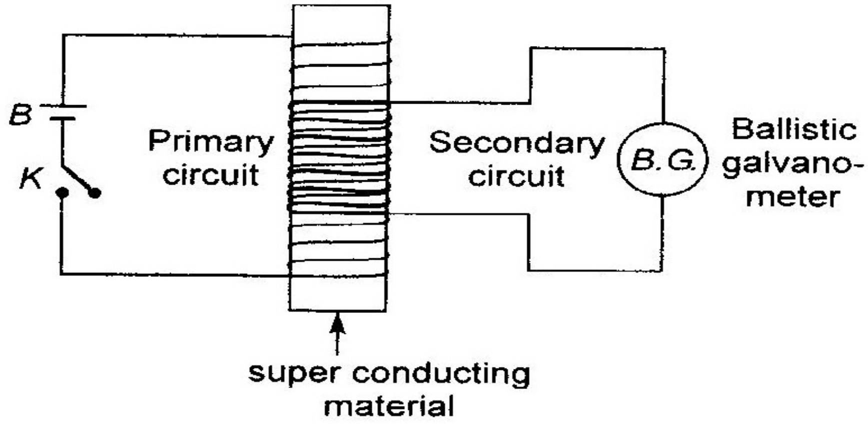
In 1933 Meissner observed that if a superconductor is cooled in a magnetic field, below critical temperature corresponding to that field, than the lines of induction are expelled from the material this effect is known as Meissner effect.

As shown in the fig magnetic lines of force pass through the superconductor when it is in the normal state. But when the specimen is cooled below its transition temperature the magnetic lines of force are expelled out of the specimen.

IMPORTANT POINTS:

(1) Meissner effect is reversible.
(2) A superconductor is a perfect dia-magnetic.
(3) The difference between a perfect conductor and a superconductor is that the former is only an ideal conductor while the later is simultaneously an ideal conductor and an ideal dia-magnet.

EXPERIMENTAL DEMONSTRATION OF MEISSNER EFFECT:



Showing the demonstration of Meissner effect.

As shown in the fig two coils are wounded on a superconductor. The superconductor is above its critical temperature. In primary circuit, there is a battery and a plug key. The secondary coil is connected to a galvanometer. When the key K is pressed, a current flow through the primary coil as a result the galvanometer shows a momentary deflection. After that the primary current becomes steady. Hence no deflection is observed in the B.G.

Now the temperature of superconductor is decreased gradually. As soon as the temperature crosses below the critical temperature, the galvanometer shows a deflection. This indicates that the flux linked with the secondary coil has changed. The change in magnetic flux is due to expulsion of magnetic flux from superconducting material. This shows the experimental demonstration of Meissner effect.

PERSISTANT CURRENT:

The resistance of the superconductor is zero at a temperature below its critical temperature. That is in a superconductor currents can flow with no appreciable loss of energy. This super current persists forever without any need for an external drilling emf.

It continues for a period of two and a half years observed by S.C. Collins. The decay time for these persistent current is certainly more than 10^5 years.

IMPORTANT POINTS:

(1) Superconductivity is destroyed by application of a sufficiently large magnetic field.
(2) If the current exceeds a critical current the superconducting state will be destroyed.

ISOTOPE EFFECT:

It was discovered that the critical temperature of the isotope of a superconducting element decrease with increasing atomic mass of the isotope. The T_c for the isotope ^{199}Hg is 4.161K and for the isotope ^{204}Hg is 4.126K. The isotope effect in its generalized form can be stated as $T_cM^{1/2}$=constant

Here M is the mass of the isotope. This isotopic effect suggests that the current carrying electrons in a superconductor do not move independently of ion lattice, instead of some low interacting with the lattice. Thus the isotopic effect revealed that electron-photon interactions are deeply involved in the phenomenon of superconductivity.

THERMODYNAMIC PROPERTIES:

Gibbs free energy G can be represented by G = U-Ts-M.B------------- (1)

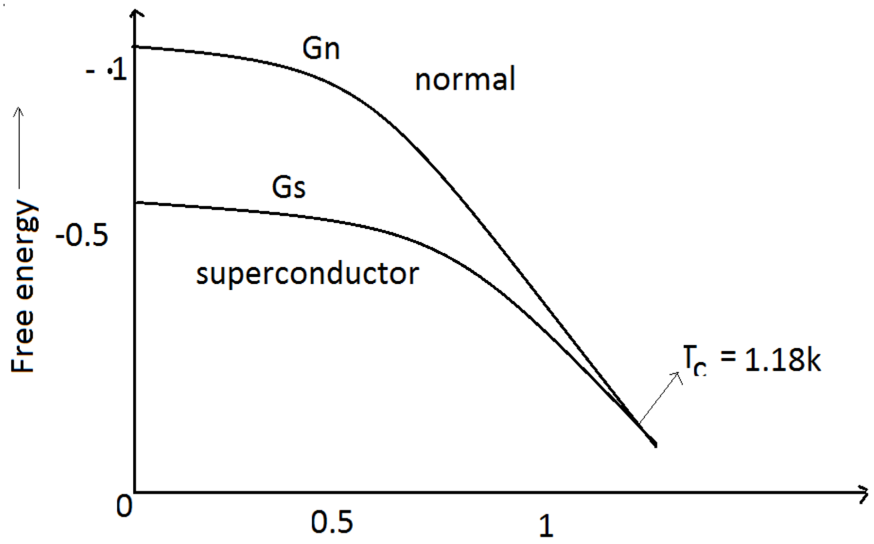
Here U = internal energy
T = temperature
S = entropy
M = magnetic moment
B = magnetic induction

If we neglect volume changes and considering magnetic wave terms then dG = -M.dB.
From the above eq. we can derive the following equation

$$G_n - G_s = \frac{B_c^{\,2}}{2\mu_o} \text{---------------------} (2)$$

Here G_n and G_s refers to Gibbs free energy of the normal and the super conducting state, B_C is the critical magnetic field.

The above relation shows that in zero magnetic field the superconducting state is lower in free energy by $\frac{B_c^{\,2}}{2\mu_o}$ per unit volume. This is shown in the fig

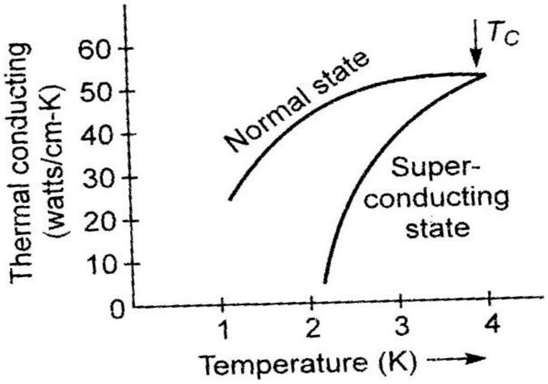


By using eqn. (2) we can estimate the critical field B_c(T) from the above graph.
The difference of entropies from eq. (2) as

$$S_n - S_s = \frac{-B_c}{\mu_o} \cdot \frac{dB_c}{dT} ------------- (3)$$

Since the slope $\frac{dB_c}{dT}$ of the critical field curve is –ve, Sn > Ss revealing that the superconducting state is more ordered state than the normal state.

THERMAL CONDUCTIVITY:

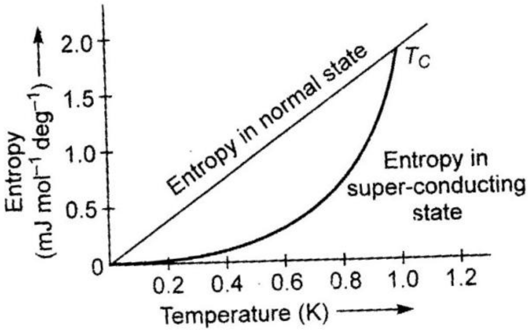


Thermal conductivity of a specimen of tin in normal and superconducting states.

The thermal conductivity of superconductors undergoes a continuous change between two phases. This is usually lower in superconducting state as shown in the fig.

ENTROPY:

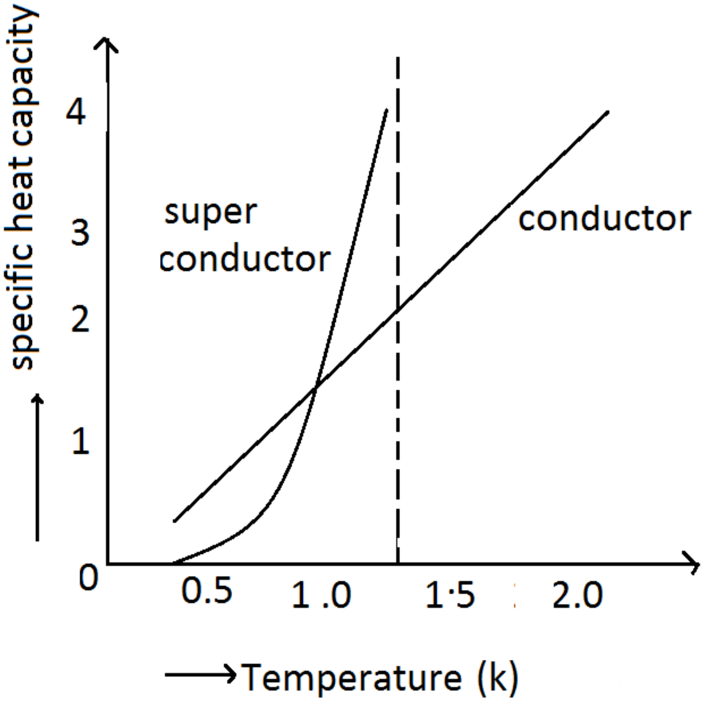
Entropy is a measure of dis-order in the system. The entropy of a superconductor decreases rapidly on cooling below the transition temperature. This shows that superconducting state is more ordered than the normal state. Below T_c, the entropy starts decreasing and the electrons which were excited in normal state start ordering in superconducting state.



Showing the change of entropy of superconductor with temperature.

SPECIFIC HEAT:

At low temperatures the specific heat C_n of a normal metal is



given by Debye’s theory as $C_n = AT + BT^3$ ------------(1)

The linear term AT is due to electronic excitations and the term T^3 are due to lattice vibrations. When the metal is in superconducting state the specific heat C_s will jump to a higher value initially at T_c and then will slowly decrease and finally off quite below the value of normal state indicating the existence of an energy gap. This is shown in the fig.

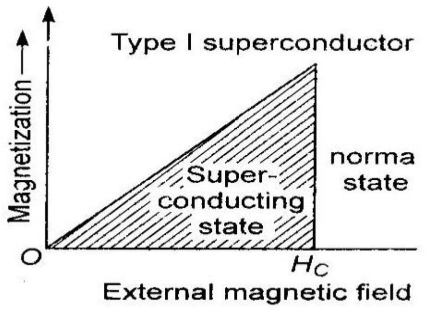
At very low temperature the specific heat falls below the normal state.

TYPE 1 AND TYPE 2 SUPERCONDUCTORS

On the basis of magnetic behavior, the superconductors are classified into the following categories.

TYPE1 SUPERCONDUCTORS (or) SOFT SUPERCONDUCTORS

TYPE2 SUPERCONDUCTORS (or) HARD SUPERCONDUCTORS



Showing magnetization curve for a type-I superconductor

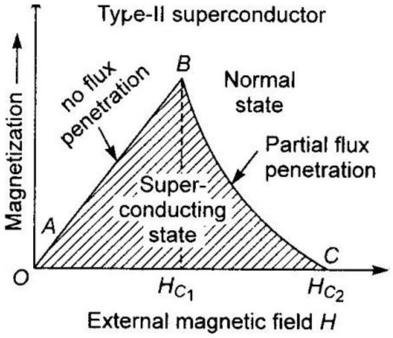
TYPE1 SUPERCONDUCTORS:

A superconductor in which the transition from superconducting to normal state in presence of magnetic field occurs sharply at the critical value H_c.

In the presence of external magnetic field $H<H_c$, type 1 superconductor in superconducting state is a perfect dia magnet. When H exceeds H_c, the superconductor enters the normal state and loses its diamagnetic property completely. In this state the magnetic flux penetrates throughout the superconductor.

TYPE2 SUPERCONDUCTORS:

Type 2 superconductor characteristic curve contain two critical magnetic fields Hc_1 and Hc_2 as shown in the fig



Showing magnetization curve for type-II superconductor.

(1) For the field strength below H_c, superconductor expels magnetic field from its body completely and behaves a perfect dia magnet. Hc_1 is called the lower critical field. The curve is represented by AB.

(2) As the magnetic field increases from Hc_1, the magnetic field lines begin to penetrate the material. The penetration increases until Hc_2 is reached. Hc_2 is called upper critical field. At Hc_2 the external field has completely penetrated into superconductor and destroys the superconductivity in the region $Hc_1$and Hc_2, the magnetic material is in a magnetically mixed state but electrically it in a superconductor.

After Hc_2 the material turns to normal state.

DEF: type 2 superconductors is one which is characterized by two electrical fields Hc1and Hc_2 and transition to normal state takes place gradually as magnetic field is increased from $Hc_1$to Hc_2.

APPLICATIONS OF SUPERCONDUCTORS:

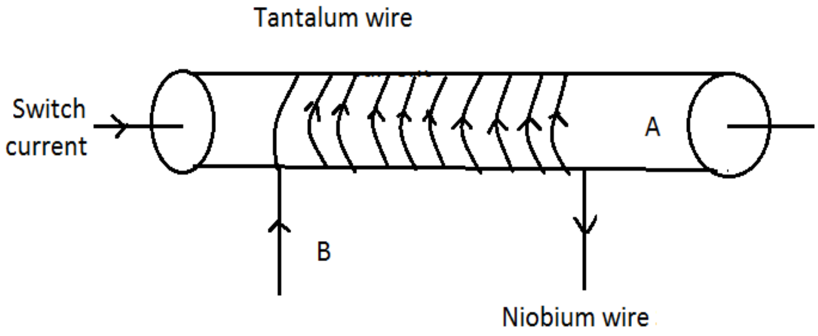
(1) POWER APPLICATIONS:

If superconductors are used in power transition, the energy loss I^2R will be eliminated and power transmission can be done at lower voltage level.

(2) SUPER CONDUCTING MAGNETS:

An electromagnet made by using coils of superconducting wires is known as superconducting magnets. The main advantage is that once the current is set up, the coil requires no source of emf to drill the current. Superconducting magnets are used in magnetic resonance imaging (MRI) technique. This technique is used to generate images of body cross-section. It is much safer than using X-rays for the some study.

(3) ELECTRICAL APPLICATIONS: CRYOTRON:



It consists of a wire of superconducting material A around which another wire of superconducting material B is wound in the form of solenoid as shown in the fig.

If a current is made to flow through Niobium wire B, then it will produce a magnetic field around wire A. if this current is enough to produce magnetic field greater than Hc_1 then tantalum become a normal conductor. Hence A is switched to its
normal state and offers high resistance to the flow of current through tantalum. Now through A is stopped.

By removing the current through niobium, the magnetic field around tantalum could be brought below Hc. The current will again flow through A.

Hence it acts as fast acting switch.

(4) MAGLEV VEHICLES:

The maglev vehicles consist of superconducting magnets built into its base. Say the vehicle runs over aluminum track in which a current is flowing. The train is set a float by magnetic deviation. This is due to enormous repulsion between two highly powerful magnetic fields, one produced by the superconducting magnet inside the train and the other due to electric currents in the aluminum track. As a result the train floats without touching the track as repulsion between superconducting magnets and the magnetic field induced in the track. A speed of 321 miles/hour was recorded in 1979 by a magnetic deviated train in Japan.

(5) VERY STRONG MAGNETIC FIELDS:

By using superconducting materials we can produce very strong magnetic fields.

GENERAL PROPERTIES OF NUCLEI

Basic ideas of nucleus -size, mass, charge density (matter energy), binding energy, angular momentum, parity, magnetic moment, electric moments. Liquid drop model and Shell model (qualitative aspects only) - Magic numbers.

INTRODUCTION:

From the scattering of α particles, Rutherford concluded that the atom of any element consists of central core called nucleus and electrons moving around it. The entire mass of the atom and positive charge is concentrated inside the nucleus.

Mass of electron=9.1×10^{-31}kg
Mass of Proton=1.673×10^{-27}kg
Mass of Neutron=1.674×10^{-27} kg

The number of protons in the nucleus is called atomic number and the sum of protons and neutrons is called as mass number. The stability of the nucleus depends upon the relative number of protons and neutrons.

BASIC PROPERTIES OF NUCLEUS:

(a) NUCLEAR MASS:

The mass of the nucleus is the sum of the masses of the neutrons and protons. It is expressed in terms of atomic mass unit

1 a.m.u=1.66×10^{-27}kg=9.315MeV

EX: mass of ${}_6C^{12}$ is 12 a.m.u

Mass of the nucleus =$Zm_p + Nm_n$ where m_p=mass of proton
Real nuclear mass< Zm_p+Nm_n m_n=mass of neutron

The difference of the mss is called mass defect. The mass defect is converted into energy according to $E=mc^2$ formula.

(b) NUCLEAR CHARGE:

The charge on the nucleus is due to protons contained in it. The charge on each proton is $+1.6\times10^{-19}$ coulomb. This is equal in magnitude to the charge of an electron. For example, a hydrogen nucleus carries a single unit charge.,

(c) NUCLEAR RADIUS:

Nuclear diameter can be determined by the experiments like scattering of fast protons or neutrons or scattering of high energy of electrons. Rutherford concluded that the distance of the closest approach of the α particle to the nucleus of the scatter can be regarded as a measure of the size of the nucleus. He found this distance of the order of 10^{-14}m. This can be determined from the empirical formula

$$r=r_oA^{1/3}$$

Where r_o=linear constant 1.4×10^{-15}m
\` A= mass of the nucleus.

EX :(1) Carbon (A=12) : $r=1.4\times10^{-15}\times(12)^{1/3}=3.21\times10^{-15}$m
(2) Copper (A=63) : $r=1.4\times10^{-15}(63)^{1/3}=5.97\times10^{-15}$m
(3) Uranium (A=238): $r=8.68\times10^{-15}$m

NUCLEAR DENSITY:

Density of Nucleus =mass of the Nucleus/volume of the Nucleus

$$= (\text{mass number x mass of proton})/\frac{4}{3}\pi r^3$$

$$= \frac{AX1.673X10^{-27}}{\frac{4}{3}\pi\left(1.5X10^{-15}A^{\frac{1}{3}}\right)^{3}}$$

$$= \frac{1.673X10^{-2}A}{14.15X10^{-45}A} = 1.18x10^{17}kg/m^{3}$$

NUCLEAR QUANTUM STATES:

Every nucleus possesses a set of quantum states in a corresponding number of discrete energy levels. Transition between different nuclear states is accompanied by the emission of γ rays.

NUCLEAR SPIN:

In atomic spectra, we found many spectral lines which have fine structure. That was explained by electron spin. Many of these fine structure lines show still finer structure called "hyper fine structure" which has been attribute to the nuclear spin.

Both the proton and the neutron have an intrinsic angular momentum generally it is called as its spin.

The magnitude of the spin angular momentum is $\frac{1}{2}\hbar$ where $\hbar = \frac{h}{2\pi}$. The nucleons [protons and neutrons] possess orbital angular momentum due to motion about the centre of the nucleus. The resultant angular momentum I of the nucleon is obtained by adding spin and orbital angular momentum of all nucleons in the nucleus.

Total angular momentum I =L+S

Where L=orbital angular momentum

S=spin angular momentum

MAGNETIC DIPOLE MOMENT OF NUCLEI (μ):

We know that the charged particle moving in a closed path produces a magnetic field. The spinning electron is associated with a magnetic dipole moment of one "Bhor magneton"

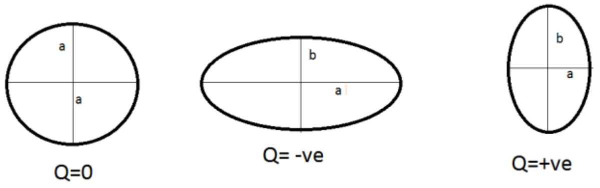
i.e., $\mu_e = \frac{e\hbar}{2m_e}$ where e is the charge m_e is the mass of the electron.

Due to proton spin, it has Magnetic dipole moment $\mu_N = \frac{e\hbar}{2m_p}$ where m_p is the proton mass and μ_N is called nuclear magneton.

Since m_p=1836xm_e so nuclear magneton $\frac{1}{1836}$ of a Bohr magneton.

Neutron is a neutral particle. So it is very difficult to understand how it can have a magnetic moment. It is found that neutron has a magnetic moment $\mu_n = -1.9128\mu_N$. To explain μ_n, it was assumed that neutron contains equal amount of +ve and –ve charges. If these charges are not uniformly distributed then a spin magnetic moment may arise. The magnetic moments of nuclei are found to vary between -2 to +4 nuclear magneton.

ELECTRIC QUADRUPOLE MOMENT (Q): Experiments show that the shape of the nucleus is not spherical but it is an ellipsoid of revolution. The derivation from the spherical symmetry is measured by a quantity called its electric quadrupole moment.



SHAPE OF NUCLEUS AND ELECTRIC QUADRUPOLE MOMENTS

If the axis of symmetry is 2a and the diameter in the perpendicular direction is 2b, the electric quadrupole moment is defined as

$$Q=\frac{2}{5}ze\left(b^2-a^2\right)$$

Where z is atomic number and Ze is total charge of nucleus.
Therefore Q = 0 for spherical shaped nucleus, Q is negative for a > b and Q is positive for b > a as shown in the above fig.

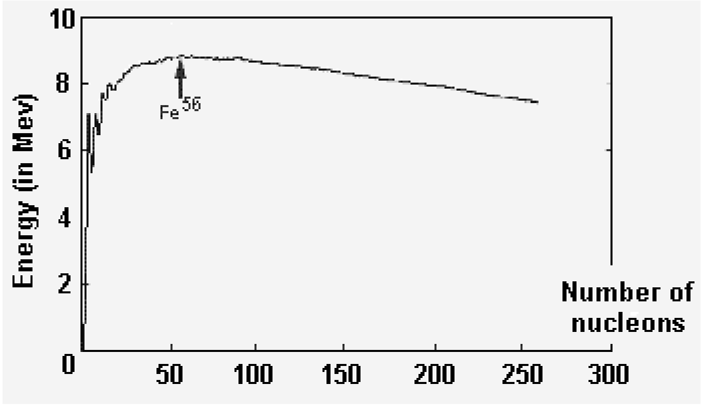
BINDING ENERGY OF NUCLEUS:

When protons and neutrons combine to form a nucleus, some of the mass (Δm) is found to be disappeared. This is known as mass defect. This Δm mass is converted into an amount of energy equal to $(\Delta m)c^2$. This energy is called the binding energy of the nucleus. To separate a stable nucleus into its constituent protons and neutrons, the minimum energy required is the binding energy. If the binding energy is large the nucleus is stable.

If M is the mass of the nucleus having Z protons and N neutrons then

$$B.E= [(Zm_p+Nm_n)-M]\,c^2$$

Where, m_p and m_n are the mass of proton and neutron respectively. If B.E<0, the nucleus is unstable and it will disintegrate by itself.



The ratio of binding energy of the nucleus and number of nucleons in it is known as B.E per nucleon. The B.E per nucleon is plotted as function of mass number A in the above fig. from the graph it is clear that nuclei of intermediate mass[A=40 to 140] are most stable nuclei. The binding energies of He^4, C^{12}, O^{16} are considerably greater than those of their neighbors.

When a heavier nucleus splits into lighter nuclei or lighter nuclei join to form heavier nucleus energy is released. The first process is called nuclear fission and the second is called nuclear fusion.

BINDING ENERGY OF DEUTRON:

The isotopes of hydrogen are ${}_1H^1$ and ${}_1H^2$. The heaviest isotope of hydrogen is called deuteron. It consists of one proton and one neutron in its nucleus.

B.E of ${}_1H^2$ =total mass of proton and neutron-actual mass of ${}_1H^2$

B.E of $_1H^2$ = (2.016490-2014104)a.m.u
=0.002386x931 MeV
=2.23 MeV

This shows that when a proton and neutron are combined to form a deuteron, energy equal to 2.23 MeV is released.

NUCLEAR MODELS:

The force that binds the proton and the neutron so securely into a tiny molecule is the strongest type of force known. This nucleus force is a short range force. To explain the nuclear properties, theoretical physicist developed various nuclear models. They are (1) liquid drop model and (2) shell model

LIQUID DROP MODEL:

This model was proposed by Neil's Bhor in 1937. According to this model

(1) The nucleus is similar to a small electrically charged liquid drop i.e., nucleus take a spherical shape for its stability.
(2) The nucleons move within this spherical enclosure like molecules in a liquid drop.
(3) The motion of nucleons within nucleus is a measure of nuclear temperature as the molecular motion of molecules in liquid is the measure of its temperature.
(4) The nucleons always remains constant distant a part and share among them the total energy of the nucleus.
(5) The nucleons inside the nucleus are attracted from all sides by neighboring nucleons while those on the surface are attracted from one side only. In this way the binding energy for the nucleons at the surface of the nucleus is smaller than the binding energy for the nucleons inside the nucleus.

In this model the nucleus is regarded as arranged to a drop of liquid. Hence this model is known as liquid drop model.

FOLLOWING ARE THE ANALOGIES BETWEEN LIQUID DROP AND A NUCLEUS:

(1) Both are spherical in nature.
(2) In both the cases, the density is independent of its volume.
(3) The molecules in liquid drop interact over short ranges and so is true for nucleons in nucleus.
(4) As the surface tension forces act on the surface of a drop similarly a potential barrier acts on the surface of nucleus.
(5) When the temperature of the molecules in a liquid drop increased, evaporation of molecules takes place. Similarly, when the nucleons in the nucleus are subjected to external energy, a compound nucleus is formed which emits nucleons almost immediately.

MERITS:

(1) It has been successfully explains nuclear reactions and nuclear fission.
(2) The calculations of atomic mass and binding energy can be done with good accuracy.

DE-MERITS:

This model fails to explain other properties like magic numbers (2, 8, 20, 28, 50, and 82,126).

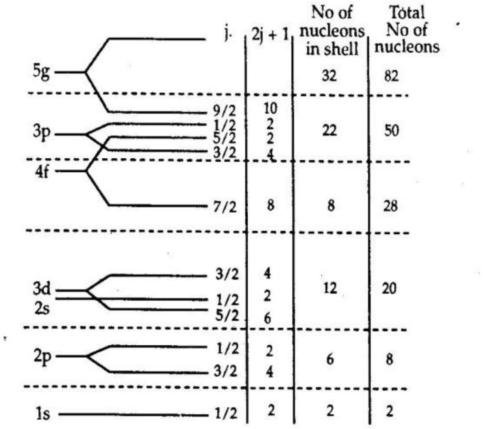
THE SHELLMODEL:

The nucleus consists of a series of protons and neutrons placed in certain discrete levels or shells just like the electrons in the discrete shells of an atom. According to Pauli's exclusion principle two protons with opposite spins and two neutrons having opposite spins are accommodated in a particular shell. In this way the first shell accommodates two protons and two neutrons and is more tightly bound than other shells.

THE CONCEPT OF THE SHELL MODEL IS BASED ON THE FOLLOWING FACT:

It has been observed that the nuclei containing protons and neutrons numbers 2, 8,20,50,82,126 called magic numbers are exceptionally stable. This shows that there are definite energy shells in the nuclei.

In 1949 Maria-Mayer and Jensen proposed that magic numbers can be obtained by assuming that there is strong interaction between the spin angular momentum S and the orbital angular momentum L of the nucleon. If we write j=*l* + s. we get two values. They are *l*+s and *l*-s. The energy depends on total angular momentum quantum number j. for each value of j there will be 2j+1 different states corresponding to different projections of j on the Z-axis. Thus there will be 2j+1nucleons in a state of given j.



As shown in the fig, the energy levels are grouped into shells such that each shell contains closely spaced energy levels and two consecutive shells are separated by a large gap. The closed shells are obtained ,whenever the number of nucleons are 2,8,20,28,50 etc… Thus magic numbers are obtained. The shell structure applies to both neutrons and protons separately.

MERITS:

This model explains the magic numbers, observed angular momenta, magnetic moments and quadrupole moments of nuclei.

DE-MERITS:

This model fails to explain the large nuclear quadrupole moment and spherical shapes of many nuclei.

Magic numbers: In nuclear physics, a **magic number** is a number of nucleons (either protons or neutrons, separately) such that they are arranged into complete shells within the atomic nucleus. The seven most widely recognized magic numbers are **2, 8, 20, 28, 50, 82, and 126** .For protons, this corresponds to the elements helium, oxygen, calcium, nickel, tin, lead and the hypothetical unbihexium, although 126 is so far only known to be a magic number for neutrons. Atomic nuclei consisting of such a magic number of nucleons have a higher average binding energy per nucleon than one would expect based upon predictions such as the semi-empirical mass formula and are hence more stable against nuclear decay.

Parity: Parity is a fundamental character of nucleus. Parity can be even or odd depending on nucleon number of the atomic nucleus.

In physics, property important in the quantum-mechanical description of a physical system. In most cases it relates to the symmetry of the wave function representing a system of fundamental particles. A parity transformation replaces such a system with a type of mirror image. Stated mathematically, the spatial coordinates describing the system are inverted through the point at the origin; that is, the coordinates x, y, and z are replaced with $-x$, $-y$, and $-z$. In general, if a system is identical to the original system after a parity transformation, the system is said to have even parity. If the final formulation is the negative of the original, its parity is odd. For either parity the physical observables, which depend on the square of the wave function, are unchanged. A complex system has an overall parity that is the product of the parities of its components.

III B.Sc. Physics Paper IV (Modern Physics)

Problems and Answers

1. **Energy of a particle at absolute temperature T is of the order of KT. Calculate the wavelength of thermal neutrons at 27^oC. Mass of neutron is 1.67×10^{-27} kg. Planck's constant $h = 6.60\times10^{-34}$ J-sec, Boltzmann constant $K = 8.6\times10^{-5}$ eV/deg.**

Ans: Given $T = 27\ ^oC = 27+273 = 300$ K

$m = 1.67\times10^{-27}$ kg

$h = 6.60\times10^{-34}$ J-sec

$K = 8.6\times10^{-5}$ eV/deg

Wavelength of particle $\lambda = \dfrac{h}{mv}$

$$= \frac{h}{\sqrt{2mE}} \quad \text{(since } v = \sqrt{\frac{2E}{m}}\text{)}$$

$$= \frac{h}{\sqrt{2mKT}} \quad \text{(since energy of particle } E = KT\text{)}$$

Substituting the values of h, m, K and T in the above equation, we get

$\lambda = 1.777$ Å

2. **Calculate the wavelength associated with an electron having K.E. equal to 1.512 MeV. Rest mass energy is 0.512 MeV.**

Ans: Given Kinetic energy K.E. = 1.512 MeV

Rest mass energy = $m_oc^2 = 0.512$ MeV

$mc^2 = m_oc^2 + \text{K.E.}$ (c = velocity of light = 3×10^8m/s)

$= 0.512 + 1.512$

$= 2.024$ MeV $= 2.024\times10^6\times1.6\times10^{-19}$ J (since 1eV = 1.6×10^{-19} J)

$$m = \frac{2.024\times1.6\times10^{-13}}{(3\times10^8)^2} = 3.598\times10^{-30} \text{ kg}$$

Rest mass $m_o = 0.512\times10^6 / (3\times10^8)^2 = 4.608\times10^{-10}$ kg

The speed of the electron is calculated from the equation

$$m=\frac{m_0}{\sqrt{1-v^2/c^2}}$$

Substituting the values of m, m_0 and c in the above equation, we get

$v = 2.902\times10^8$ m/s

Wavelength associated with the electron

$\lambda=\frac{h}{mv}$, where h = Planck's constant = 6.60×10^{-34} J-sec

Inserting the values of h, m and v, we get

wavelength **λ = 0.00634 Å**

3. **A nucleus is confined to nucleus of radius 5×10^{-15} m. Calculate the minimum uncertainty in the momentum of the nucleon.**

Ans: Given uncertainty in position = $\Delta x = 5\times10^{-15}$ m.

According to Heisenberg uncertainty relation

$\Delta x.\Delta P_x \approx \frac{h}{2\pi}$, where h = Planck's constant = 6.60×10^{-34} J-sec

Substituting the values of h and Δx, we get

Uncertainty in momentum $\mathbf{\Delta P_x = 1.05\times10^{-20}}$**kg-m/s**

4. **A microscope using photons is employed to locate an electron in an atom to within a distance of 0.1 Å. What is the uncertainty in velocity?**

Ans: Given uncertainty in position = Δx = 0.1 Å = 0.1×10^{-10} m.

According to Heisenberg uncertainty relation

$\Delta x.\Delta P_x \approx h$, where h = Planck's constant = 6.60×10^{-34} J-sec

Substituting the values of h and Δx, we get

Uncertainty in momentum $\Delta P_x = m_0.\Delta v$

Mass of electron $m_0 = 9.1\times10^{-31}$ kg

Therefore, $\Delta x.m_0\Delta v \approx h$,

$$\Delta v = \frac{h}{m_0\Delta x} = \frac{6.60\times10^{-34}}{9.1\times10^{-31}\times0.1\times10^{-10}}$$

$$= 7.28\times10^7 \text{ m/s}$$

5. **Calculate the longest wavelength that can be analysed by rock salt crystal of spacing 2.82 Å in the first order.**

Ans: Given interplanar spacing d = 2.82 Å.

According to Bragg's law

$$n\lambda = 2d\sin\theta$$

where, n = order of diffraction = 1 for first order

λ = wavelength

θ = Bragg's angle

$(\sin\theta)_{max} = 1$, which means $\theta_{max} = 90^o$

Substituting the values of d, n and θ_{max}, we get $\boldsymbol{\lambda_{max} = 5.64}$ **Å**

6. **The radius of Ho^{165} is 7.731 fermi. Find the radius of He^4.**

Ans: Given radius of Ho^{165} r_1 = 7.731 fermi

Mass number of Ho^{165} $A_1 = 165$

Mass numbers and radii are related by the equation

$$r = r_0 A^{1/3}$$

Let r_2 be the radius and A_2 be the mass number of He^4

Mass number of He^4 $A_2 = 4$

Then, $r_1 = r_o A_1^{1/3}$

$r_2 = r_o A_2^{1/3}$

And $\frac{r_1}{r_2} = (\frac{A_1}{A_2})^{1/3}$

Substituting the values of r_1 , A_1 and A_2 in the above equation, we get

r_2 **= 2.238 fermi**

7. **X-rays of wavelength 1.54 Å are diffracted by a crystal with first maximum occurring at 11^o. What is the separation between the lattice planes?**

Ans: Given wavelength of X-rays λ = 1.54 Å

Bragg's angle $\theta = 11^o$

Order of diffraction $n = 1$

According to Bragg's law, $n\lambda = 2d\sin\theta$

Substituting the values of λ, θ and n, we get

Interplanar spacing $\boldsymbol{d}$ **=4.035 Å**

8. **A superconductor material has a critical temperature 3.7 K. Critical magnetic field of 0.306 tesla at 0 K. Find the critical field at 2 K.**

Ans: Given critical temperature T_c = 3.7 K

Critical field at 0 K H_c (0) = 0.306 tesla

At T = 2 K, the critical field is Given by

$$H_c(T) = H_c(0).[1-(T/T_c)^2]$$
$$= 0.0306.[1-(2/3.7)^2]$$
$$= \mathbf{0.0217\ tesla}$$

9. **If K_α-radiation of Mo (z = 42) has a wavelength of 0.75 Å, calculate the corresponding wavelength of radiation coming out of Co (z = 27).**

Ans: Given atomic number of Mo = $z_1 = 42$

Atomic number of Co = $z_2 = 27$

Wavelength of K_α-radiation of Mo = $\lambda_1 = 0.75$ Å

Wavelengths and atomic numbers are related by the equation

$$\frac{\lambda_1}{\lambda_2} = \frac{(z_2 - 1)^2}{(z_1 - 1)^2}$$

Substituting the values of z_1, z_2 and λ_1 in the above equation, we get

Wavelength of radiation coming out of Co, $\boldsymbol{\lambda_2 = 1.865}$ **Å**

10. The exciting line in an experiment is at 5460 Å and the stokes line at 5520 Å. Find the wavelength of anti-stokes line.

Ans: Given wavelength of exciting line $\lambda_0 = 5460$ Å

Wavelength of stokes line $\lambda_s = 5520$ Å

Let wave numbers of stokes, anti-stokes and exciting line are ν_s, ν_{as} and ν_0 respectively. Let $\Delta\nu$ be the Raman shift in wave numbers.

Wave number of anti-stokes line is Given by

$\nu_{as} = \nu_0 + \Delta\nu$

where, Raman shift $\Delta\nu = \nu_0 - \nu_s$

Therefore, $\nu_{as} = \nu_0 + \nu_0 - \nu_s = 2\nu_0 - \nu_s$

$$= 2\times\frac{1}{\lambda_0} - \frac{1}{\lambda_s}$$

$$= \frac{2}{5460} - \frac{1}{5520}$$

$$= \frac{2\times 5520 - 5460}{5460\times 5520}$$

Wavelength of anti-stokes line $\lambda_{as} = \dfrac{1}{\nu_{as}} = \dfrac{5520\times 5460}{2\times 5520 - 5460} =$ **5401 Å**

11. The wavelength of H_β line is at 4861 Å. Calculate the Rydberg constant.

Ans: Given wavelength of H_β line $\lambda = 4861$ Å

Wavelength and Rydberg constant for H_β line are related by the equation

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = R\frac{3}{16}$$

Therefore, substituting the value of λ in the above equation, we get

Rydberg constant $\boldsymbol{R = 1.097\times 10^{7}\ \mathrm{m^{-1}}}$

12. What is the lowest energy that a neutron (mass = 1.67×10^{-27}kg) can have if confined to move along the edge of an impenetrable box of length 10^{-14} m? ($h = 6.63\times10^{-34}$J-S)

Ans: Given mass of neutron $m = 1.67\times10^{-27}$ kg

length of impenetrable box $a = 10^{-14}$ m

Planck's constant $h = 6.63\times10^{-34}$J-S

Energy of discrete energy levels in an impenetrable box is given by

$$E_n = \frac{n^2h^2}{8ma^2}$$

For lowest energy level, $n = 1$

Substituting the values of n, h, m and a, we get

$$\boldsymbol{E_1 = 3.29\times10^{-13}\ \text{J} = 2.056\ \text{MeV}}$$

13. Find the binding energy of an α-particle from the following data:

Mass of Proton = 1.007277 amu.
Mass of He nucleus = 4.001265 amu.
Mass of Neutron = 1.008666 amu.
Take 1 amu = 931.4 MeV.

Ans: Given Mass of Proton m_p = 1.007277 amu.

Mass of He nucleus m_{He} = 4.001265 amu.

Mass of Neutron m_n = 1.008666 amu.

In an Helium nucleus, there are 2 protons and 2 neutrons.

Therefore, mass defect in electron volts Δm = Binding energy (B.E.)

$$= \left[\left(2m_p + 2m_n\right) - m_{He}\right]$$

Substituting the values of m_p, m_{He}, m_n in the above equation, we get the value of

B.E. = 28.52 MeV

14. How many electrons, protons and neutrons are there in 14 grams of $_6C^{14}$? (Avogadro's Number = 6.023×10^{23})

Ans: Given mass of $_6C^{14}$ = 14 grams = 1 mole

Number of atoms present in 1 mole of a substance = Avogadro's Number
= 6.023×10^{23}

Therefore, number of atoms present in 14 grams of $_6C^{14}$ = 6.023×10^{23}

Number of protons in one $_6C^{14}$ atom = atomic number = 6
Number of electrons in one $_6C^{14}$ atom = 6
Number of neutrons in one $_6C^{14}$ atom = 14 – 6 = 8

Therefore, **number of protons in 14 grams of $_6C^{14}$ atom = $6 \times 6.023\times10^{23}$**
number of electrons in 14 grams of $_6C^{14}$ atom = $6 \times 6.023\times10^{23}$
number of neutrons in 14 grams of $_6C^{14}$ atom = $8 \times 6.023\times10^{23}$

15. How many kilowatt hours of energy will be released by the complete fission of 1 gram of U^{235}? (Avogadro's Number = 6.023×10^{23})

Ans: Given mass of U^{235} = 1gram

Mass number $A = 235$

Energy released in the fission = number of nuclei in 1 gram × energy released per fission

Therefore, energy released = $\frac{1\times 6.023\times 10^{23}}{235}\times 200\,\text{MeV}$

$= 5.126\times 10^{23}$ MeV

$= 5.126\times 10^{23} \times 10^{6}\times 1.6\times 10^{-19}$ J (1eV = 1.6×10^{-19} J)

$= \frac{5.126\times 10^{23}\times 10^{6}\times 1.6\times 10^{-19}}{3.6\times 10^{6}}$

$= 22.782\times 10^{3}$ kWH

16. Electrons are accelerated to 344 volts and reflected from a crystal. The first reflection maxima occurs, when glancing angle is 60^{o}. Determine interplanar spacing of the crystal.

$h = 6.60\times 10^{-34}$ J-S

Electron charge $e = 1.6\times 10^{-19}$ col.

Mass of electron = 9.1×10^{-31} kg

Ans: Given Acceleration voltage $V = 344$ V

Bragg angle $\theta = 60^{o}$

Mass of electron $m_e = 9.1\times 10^{-31}$ kg

Planck's constant $h = 6.60\times 10^{-34}$ J-S

Wavelength of electrons $\lambda = \frac{h}{\sqrt{2m_e eV}}$

Substituting the values of h, m_e, e, and V, we get the value of λ.

According to Bragg's law

$$2d\sin\theta = n\lambda$$

Where, d = interplanar spacing

For first maxima $n = 1$

Substituting the values of θ, n and λ, we get

Interplanar spacing $d = 0.381$ Å

17. What are the possible quantum numbers (s, l, j) of an atom in first two shells ($n = 1$, $n = 2$)?

Ans: i) for shell $n = 1$,

$l = 0$, $s = 1/2$ and $j = l \pm s = 0 \pm ½$

$= ½$

Therefore, possible values of (s, l, j) are **(1/2, 0, 1/2)**

ii) for shell $n = 2$

$l = 0$ and 1, $s = 1/2$ and $j = l \pm s = 0 \pm 1/2$ and $1 \pm 1/2$

= 1/2 and 3/2, ½ (three states are possible)

Therefore, the possible quantum numbers (s, l, j) are

(1/2, 0, 1/2), (1/2, 1, 3/2) and (1/2, 1, 1/2)

18. What is the Dowin-Haunt's limit of X-ray tube working at 40 kV?

Ans: Given operation voltage = 40 kV = 40×10^3 V

According to Dowin-Haunt's law, $\lambda_{\min} = \frac{hc}{Ve}$

Where, h is Planck's constant = 6.61×10^{-34} J-S

e = charge of electron = 1.6×10^{-19} c

and c = velocity of light = 3×10^8 ms^{-1}

Therefore, $\lambda_{\min} = \frac{12400\times10^{-10}}{V(volts)} = \frac{12400\times10^{-10}}{40\times10^3}$ = 310×10^{-13} m

= **0.310 Å**

19. Work function of Tungsten is 4.52 eV. Calculate the maximum wavelength of emitted photoelectrons.

Ans: Given work function of Tungsten ω_0 = 4.52 eV

Work function $\omega_0 = h\nu_0 = \frac{hc}{\lambda_0}$

Where, v_0 is frequency, λ_0 is wavelength of emitted photoelectron, c is velocity of light and h is Planck's constant.

Substituting the values of ω_0, h and c, we get

$\boldsymbol{\lambda_0 = 2.747\times10^{-7}}$ **m = 2747 Å**

20. Calculate the mass number (A) of a nucleus whose radius (R) is 2.72 fermi. (Given R_0 = 1.2 fermi)

Ans: Given radius of the nucleus R = 2.72 fermi

R_0 = 1.2 fermi

Let mass number be A.

Mass numbers and radii are related by the equation

$$R = R_0 A^{1/3}$$

Substituting the values of R_0 and R, we get mass number ***A* = 12**

21. Calculate the interplanar spacing for (111) plane in a simple cubic lattice where lattice constant is 4×10^{-10} m.

Ans: Given the plane ($h\ k\ l$) = (111) i.e., $h = 1$, $k = 1$, $l = 1$

Lattice constant $a = 4\times10^{-10}$ m

The relation between lattice constant a and interplanar spacing d is given by

$$a = \frac{d}{\sqrt{h^2+k^2+l^2}}$$

Substituting the values of a and ($h\ k\ l$), we get the interplanar spacing **$d = 4\sqrt{3}$ Å**

22. The spacing between the planes of NaCl crystal is 2.81 Å. It is found that the first-order Bragg reflection occurs at angle of 10°. What is the wavelength of X-rays?

Ans: Given interplanar spacing d = 2.81 Å

Bragg's angle $\theta = 10^\circ$

Order of diffraction n = 1 (first order)

According to Bragg's law, $n\lambda = 2d\sin\theta$

Substituting the values of d, θ and n, we get

Wavelength of X-rays **λ = 0.976 Å**

23. Deduce the wavelength of K_α-line for an atom of $z = 92$. Given $R = 1.1\times10^5$ cm^{-1}

Ans: For K_α-line,

$$\frac{1}{\lambda_{K_\alpha}} = \frac{3}{4}R(z-1)^2$$

Substituting the values of R and z, we get λ_{K_α} **= 0.1464×10^{-8} cm = 0.1464 Å**

24. In Uranium (z = 92), the K-absorption edge is 0.107 Å and K_α line is 0.126 Å. Determine the wavelength of L-absorption edge.

Ans: Given λ_k = 0.107 Å and λ_{K_α} = 0.126 Å

$$E_L = -\frac{ch}{\lambda_k} = -\frac{12.4}{0.107}\text{keV} = -115.9 \text{ keV}$$

And $E_L - E_K = \frac{ch}{\lambda_{K_\alpha}} = -\frac{12.4}{0.126}$ keV

$$\therefore E_L = -17.5 \text{ keV}$$

$$\therefore \lambda_L = \frac{ch}{E_L} = \frac{12.4}{17.5} \text{ Å} = \mathbf{0.709\ Å}$$

25. Calculate the rotational energy level of HCl molecule. Given bond length r = 1.3Å

Ans: Given bond length r = 1.3Å

The rotational energy is Given by,

$E_r = \dfrac{h^2}{8\pi^2 I} J(J+1)$, where I is moment of inertia, J = rotational quantum number and h is Planck's constant.

For $J = 1$, $E_r = \dfrac{h^2}{8\pi^2 I} \times 2$

Moment of inertia $I = Mr^2$, where M is the mass of HCl molecule.

$$M = M_H + M_{Cl} = 1 + 35 = 36$$

Hence, $E_r = \dfrac{2h^2}{8\pi^2 Mr^2}$

Substituting the values of Planck's constant (h), mass M and radius r, we get the desired value of energy.

26. The work function for sodium is 2.27 eV. What is the velocity of electrons emitted by light of wavelength 4000 Å. (Mass of electron is 9.1×10^{-31}kg; *h* = 6.62×10^{-34}J-S)

Ans: Given work function for sodium = 2.27 eV

Mass of electron is 9.1×10^{-31} kg

Planck's constant $h = 6.62\times10^{-34}$ J-S

We know that,

$$h\nu = \omega_0 + \frac{1}{2}mv^2$$

$$\therefore (h\nu - \omega_0) = \frac{1}{2}mv^2 = \frac{hc}{\lambda} - \omega_0$$

$$= \frac{6.62\times10^{-34}\times3\times10^{8}}{4000\times10^{-10}} - 2.27(1.6\times10-^{19})$$

$$= 1.333\times10^{-19} \text{ J}$$

$$\therefore v = \sqrt{\frac{2\times1.33\times10^{-19}}{9.1\times10^{-31}}} = 5.35\times10^{5}\ \text{ms}^{-1}$$

27. The work function of Na is 2.3 eV. Does the sodium show photoelectric effect for orange light with λ = 6800 Å.

Ans: Given work function of Na = 2.3 eV

Wavelength of orange light λ = 6800 Å

Work function $\omega_0 = h\nu_0 = \dfrac{hc}{\lambda_0}$

$$\Rightarrow \lambda_0 = \frac{hc}{\omega_0} = \frac{6.626\times10^{-34}\times3\times10^{8}}{2.3\times1.6\times10^{-19}}$$

Therefore, $\boldsymbol{\lambda_0}$ **= 5400 Å**

This is the maximum wavelength, which can produce photoelectric emission from sodium surface. But, the wavelength of Orange light is 6800 Å (given). Hence, this wavelength **can not produce photoelectric effect** from sodium surface.

28. Find the de-Broglie wavelength of a neutron whose energy is 12.8 MeV. Given m_n = 1.675×10^{-27} kg

Ans: Given energy of neutron = 12.8 MeV

Mass of neutron $m_n = 1.675\times10^{-27}$ kg

Kinetic energy K.E. = $\frac{1}{2} m_n v^2$ = 12.8 MeV = $12.8\times1.6\times10^{-19}$ J

$\therefore$ velocity $v = 1.564\times10^{7}$ ms^{-1}

De-Broglie wavelength $\lambda = \dfrac{h}{mv} = \dfrac{6.625\times10^{-34}}{(1.675\times10^{-27}\times1.564\times10^{7})}$

Therefore, $\boldsymbol{\lambda = 2.529\times10^{-20}}$ **m**

29. An electron has a speed of 600 ms^{-1} with an accuracy of 0.005%. Calculate the certainty with which we can locate the position of the electron.

Ans: Momentum of electron $P = mv = 9.1\times10^{-31}\times 600$ kgms^{-1}

$$\Delta p_x = \left(\frac{0.005}{100}\right) mv = 5\times10^{-5}\times9.1\times10^{-31}\times600$$

$$\Delta p_x.\Delta x \approx h$$

$$\therefore \Delta x = \frac{h}{\Delta p_x} = \frac{6.6\times10^{-34}}{5\times10^{-5}\times9.1\times10^{-31}\times600} \text{ m}$$

Δx = 0.02254 m

30. An electron is confined to a one dimensional potential box of width 1 Å. Calculate the first three energy levels.

Ans: The energy of a particle in an infinite one-dimensional potential well of width 'a' is Given by,

$$E_n = \frac{n^2h^2}{8ma^2}, \quad n = 1, 2, 3, \ldots$$

(1) $n = 1$; a = 1 Å = 10^{-10} m

$$E_n = \frac{\left(6.62\times10^{-34}\right)^2\times(1)^2}{8\times9.11\times10^{-31}\times(10^{-10})^2}$$

= 9.1×10^{-19} J

= **5.68 eV**

(2) $E_2 = 4\,E_1$ = 4 (5.68) = **22.7 eV**

(3) $E_3 = 9\,E_1$ = 9 (5.68) = **51.12 eV**

31. A G.M. Counter collects 10^8 electrons/discharge. Counter rate is 500 counts/min. What will be the average current in the circuit?

Ans: Counting rate = 500 counts/min

Total number of electrons collected in 1 min. = $500\times10^{8} = 5\times10^{10}$

Therefore, Charge/min. = $ne = 5\times10^{10}\times1.6\times10^{-19} = 8\times10^{-9}$

Total charge per second = $8\times10^{-9}/60 = 1.33\times10^{-10}$ A

Hence, average current in the circuit = **1.33×10^{-10} A**

32. In a crystal, lattice plane cuts the intercepts 2*a*, 3*b* and 6*c* along the three axes. Find the Miller indices of the given plane.

Ans: Here, $pa : qb : rc = 2a : 3b : 6c$

Where, *a*, *b* and *c* are primitive vectors of the unit cell

Therefore, p: q: r = 2: 3: 6

Taking reciprocals 1/p : 1/q : 1/r = 1/2 : 1/3: 1/6 = 3: 2: 1

Therefore, *h* : *k* : *l* = 3 : 2 : 1

Miller indices of the plane **(*h* *k* *l*) = (3 2 1)**